

SCHOOL
OF MATH

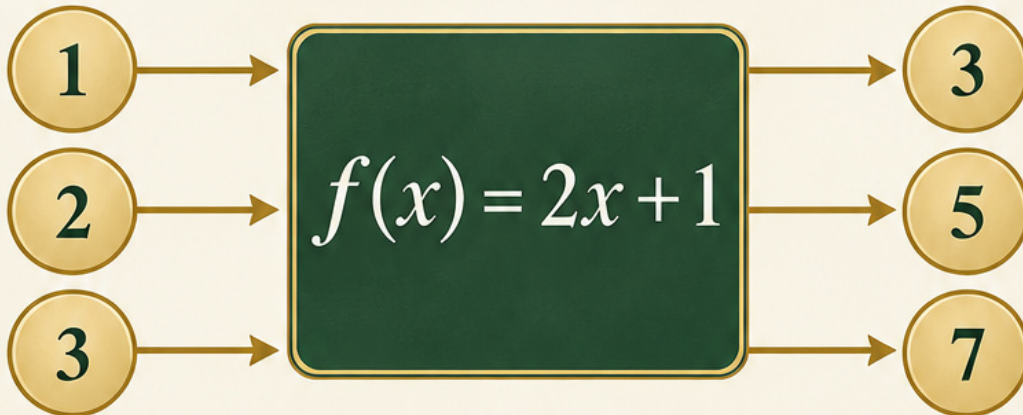
CLASS 15

FUNCTIONS

Domain, Range & Function Notation

DOMAIN

RANGE



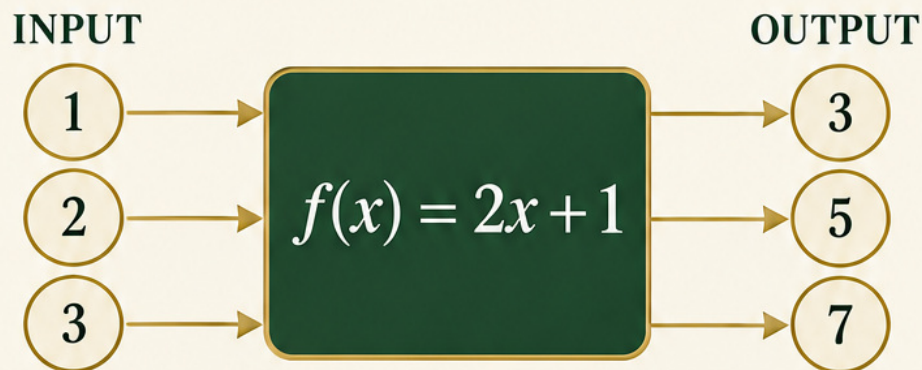
$$f(2) = 5$$

Identify • Interpret • Evaluate

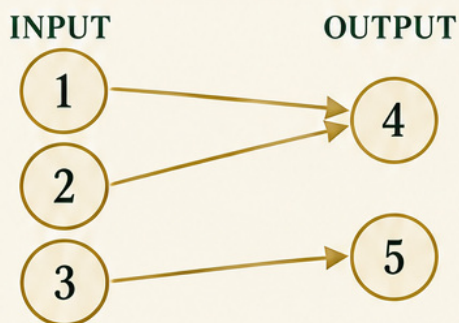
Connecting inputs, rules, and outputs

1. Understanding Functions

A **function** assigns exactly one output to each input.

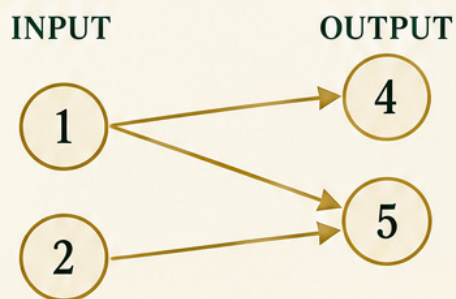


FUNCTION



Different inputs may share an output.

NOT A FUNCTION



One input cannot have two outputs.

One input. *Exactly* one output.

Image 1 practice: Understanding functions

Q1. A machine multiplies each input by 4 and subtracts 3. Find the outputs for inputs 0, 2, and 5. Does this rule define a function?

Q2. Does the relation $(2, 7)$, $(4, 7)$, $(6, 7)$ define a function? Explain.

Q3. Does the relation $(3, 8)$, $(3, 10)$, $(5, 12)$ define a function? Explain.

Q4. A rule assigns every real-number input the output 9. Is it a function? Explain.

Q5. A relation contains $(-2, 6)$ and $(1, 4)$. Which pair can be added while keeping a function: A: $(-2, 8)$ or B: $(3, 6)$? Explain.

2. Identifying Functions

Check that each input has exactly one output.

1 ORDERED PAIRS

Function

$\{(1, 4), (2, 4), (3, 5)\}$

Each input appears exactly once
and has one output.

Not a function

$\{(1, 4), (1, 5), (2, 6)\}$

The input 1 has two different
outputs.

2 TABLES

Function

x	y
1	4
2	4
3	5

Each input appears exactly once
and has one output.

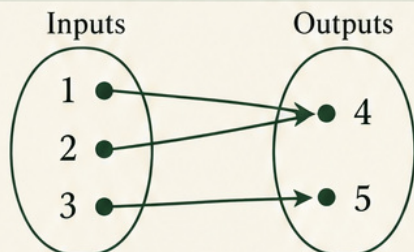
Not a function

x	y
1	4
1	5
2	6

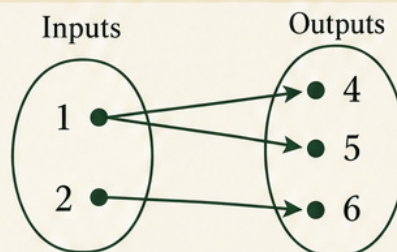
The input 1 has two different
outputs.

3 MAPPING DIAGRAMS

Function



Not a function



KEY TAKEAWAY

Repeated outputs are allowed.
Different outputs for the same input are not.

Image 2 practice: Identifying functions

Q6. Does the relation $(-3, 2)$, $(0, 2)$, $(4, 8)$ define a function? Explain.

Q7. Does the relation $(2, -1)$, $(5, 3)$, $(2, 4)$ define a function? Explain.

Q8. A table has rows $(1, 5)$, $(1, 5)$, and $(4, 9)$. Does it represent a function?

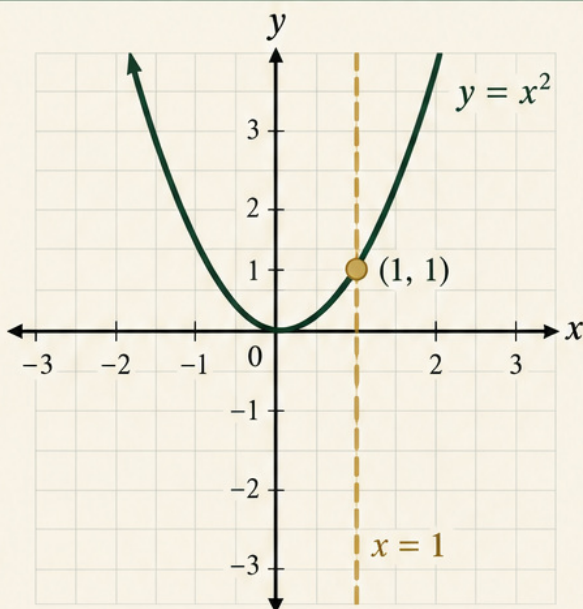
Q9. The stated domain is $\{2, 4, 6\}$. A mapping has arrows $2 \rightarrow 8$ and $4 \rightarrow 8$, but no arrow from 6. Is it a function on that stated domain?

Q10. A relation contains $(-1, 3)$, $(2, 7)$, and $(k, 9)$. Which listed values of k keep it a function: -1 , 0 , 2 , or 5 ?

3. The Vertical Line Test

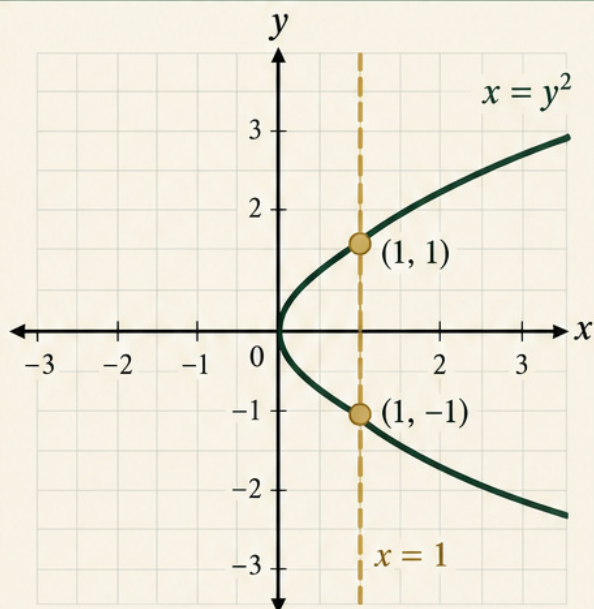
A graph represents a function if every vertical line intersects it at most once.

PASSES: A FUNCTION



One intersection for each input.

FAILS: NOT A FUNCTION



One input has two outputs.

HOW TO USE THE TEST

- 1 Imagine a vertical line moving across the graph.
- 2 Count its intersections with the graph.
- 3 If it ever meets the graph more than once, the graph is not a function.

Zero intersections are allowed: that x -value is outside the domain.

Image 3 practice: The vertical line test

Q11. Sketch $y = x + 2$. Does it represent y as a function of x ?

Q12. Sketch $y = x^2 - 2$. Does it pass the vertical line test?

Q13. Consider the circle $x^2 + y^2 = 4$. Use the vertical line $x = 0$ to decide whether y is a function of x .

Q14. Sketch the vertical line $x = -1$. Is y a function of x ?

Q15. Sketch the horizontal line $y = 3$. Does it pass the vertical line test? Explain why the repeated output is allowed.

4. Finding the Domain

The **domain** is the set of all allowed input values.

1 FROM ORDERED PAIRS

$$\{(-2, 5), (0, 1), (3, 5)\}$$

$$\text{Domain} = \{-2, 0, 3\}$$

Collect the x -values.
List each value once.

2 FROM A TABLE

x	y
-1	4
0	2
2	0

$$\text{Domain} = \{-1, 0, 2\}$$

3 FROM A RULE

$$f(x) = 2x + 1$$

Domain: all real numbers.

$$g(x) = \frac{1}{x - 3}$$

Domain: all real numbers except 3.
A denominator cannot be zero.

4 SQUARE ROOT RESTRICTION

$$h(x) = \sqrt{x - 2}$$

$$x - 2 \geq 0 \rightarrow x \geq 2$$

$$\text{Domain: } [2, \infty)$$

For real outputs, the expression
under a square root must be
nonnegative.

In a real-world problem, the context may further limit the inputs.

Image 4 practice: Finding the domain

Q16. Find the domain of the relation $(-4, 6), (1, 3), (5, 6)$.

Q17. Find the domain of $f(x) = 5x - 8$ when no other restriction is given.

Q18. Find the real-number domain of $g(x) = \frac{2}{x+4}$.

Q19. Find the real-number domain of $h(x) = \sqrt{x-5}$. Write an inequality and an interval.

Q20. A class can order from 0 through 6 whole workbooks. If n is the number ordered, give the domain of its cost function.

5. Finding the Range

The **range** is the set of all output values a function actually produces.

1 FROM ORDERED PAIRS

$\{(-2, 5), (0, 1), (3, 5)\}$

Range = $\{1, 5\}$

Collect the y-values.
List each value once.

2 FROM A TABLE

x	y
-1	4
0	2
2	0

Range = $\{0, 2, 4\}$

3 FROM A FUNCTION RULE

$$f(x) = x^2$$

Domain: all real numbers

A square is never negative.

Minimum output: 0

Range: $[0, \infty)$

4 THE DOMAIN MATTERS

$$g(x) = 2x + 1$$

Domain = $\{0, 1, 2\}$

$$g(0) = 1$$

$$g(1) = 3$$

$$g(2) = 5$$

Range = $\{1, 3, 5\}$

Domain $\rightarrow$ inputs Range $\rightarrow$ outputs

The range depends on the function rule and its domain.

Image 5 practice: Finding the range

Q21. Find the range of the relation $(-2, 7), (0, 3), (4, 7), (6, -1)$.

Q22. For $f(x) = 3x - 2$ with domain $\{-1, 0, 2\}$, find the range.

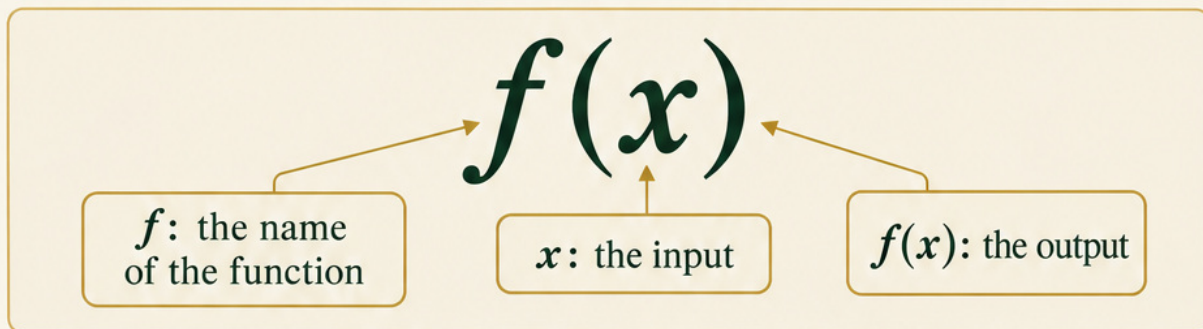
Q23. Find the range of $g(x) = x^2 + 4$ on all real inputs.

Q24. Find the range of $h(x) = 9 - x^2$ on all real inputs.

Q25. Find the range of $p(x) = x^2$ when the domain is $\{-3, 0, 3\}$.

6. Understanding Function Notation

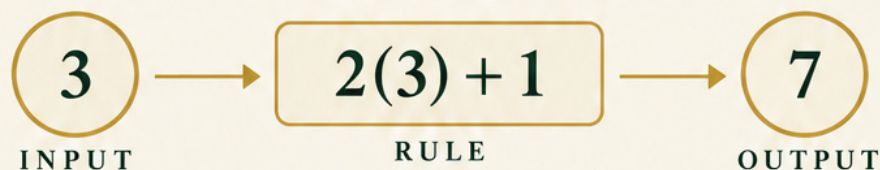
$f(x)$ means the output of function f when the input is x .



Read $f(x)$ as “ f of x .”

$$f(x) = 2x + 1$$

The rule: multiply the input by 2, then add 1.



$$f(3) = 7$$

When the input is 3, the output is 7.

SAME FUNCTION, DIFFERENT INPUTS

$$\begin{aligned} f(0) &= 1 \\ f(2) &= 5 \\ f(-1) &= -1 \end{aligned}$$

CONNECTING NOTATIONS

$$\begin{aligned} y &= 2x + 1 \\ f(x) &= 2x + 1 \end{aligned}$$

Both describe the same input-output rule.

$f(x)$ does **not** mean $f \times x$.

Image 6 practice: Understanding function notation

Q26. The statement $g(4) = 11$ is given. Identify the input, the output, and the corresponding ordered pair.

Q27. Write “function p assigns input -2 the output 7 ” using function notation.

Q28. A student says $h(3)$ means $h \times 3$. Explain the mistake.

Q29. Let $f(x) = 2x + 5$. Find $f(3)$, then solve $f(x) = 3$. Explain the difference.

Q30. Temperature is a function of time: $T(2) = 18$. Time is measured in hours after noon and temperature in degrees Celsius. Interpret the statement.

7. Evaluating Functions

Replace every x in the rule with the given input, then simplify.



POSITIVE INPUT

$$f(x) = 3x - 2$$

Find $f(4)$:

$$\begin{aligned} f(4) &= 3(4) - 2 \\ &= 12 - 2 \\ &= \mathbf{10} \end{aligned}$$

NEGATIVE INPUT

$$g(x) = x^2 + 2x$$

Find $g(-3)$:

$$\begin{aligned} g(-3) &= (-3)^2 + 2(-3) \\ &= 9 - 6 \\ &= \mathbf{3} \end{aligned}$$

FRACTION INPUT

$$h(x) = 4x + 1$$

Find $h\left(\frac{1}{2}\right)$:

$$\begin{aligned} h\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right) + 1 \\ &= 2 + 1 \\ &= \mathbf{3} \end{aligned}$$

EXPRESSION INPUT

$$f(x) = 3x - 2$$

Find $f(a + 1)$:

$$\begin{aligned} f(a + 1) &= 3(a + 1) - 2 \\ &= 3a + 3 - 2 \\ &= \mathbf{3a + 1} \end{aligned}$$

Use parentheses when substituting negative numbers or expressions.

Check that the input belongs to the domain.

Image 7 practice: Evaluating functions

Q31. For $f(x) = 4x - 7$, find $f(5)$.

Q32. For $g(x) = x^2 - 3x$, find $g(-2)$.

Q33. For $h(x) = 6x + 2$, find $h(\frac{1}{3})$.

Q34. For $p(x) = 2x + 3$, find $p(a - 2)$.

Q35. For $r(x) = \frac{10}{x-2}$, find $r(7)$. Is $r(2)$ defined?

8. Functions in Real Life

Write a rule. Define the inputs. Interpret the outputs.

MUSEUM VISIT

A group pays a \$20 booking fee plus \$8 per student. The group may include 1 to 5 students.



$$C(n) = 20 + 8n$$

n = number of students

$C(n)$ = total cost in dollars

For 3 students:

$$C(3) = 20 + 8(3) = 44$$

A group of 3 students pays \$44.

Students, n	Total cost, $C(n)$
1	\$28
2	\$36
3	\$44
4	\$52
5	\$60

DOMAIN

$\{1, 2, 3, 4, 5\}$

Allowed numbers of students.

RANGE

$\{\$28, \$36, \$44, \$52, \$60\}$

Possible total costs.

Context matters: the student count must be a whole number from 1 to 5.

Each allowed student count gives exactly one total cost.

Image 8 practice: Functions in real life

Q36. A workshop charges a \$12 setup fee plus \$5 per student. Groups have 1 to 4 students. Write $C(n)$, state the domain, and find $C(3)$.

Q37. A taxi model is $C(d) = 6 + 2.5d$ dollars for distances d from 0 through 8 miles, including fractional miles. Find the domain and range.

Q38. A store sells only packs of 4 pencils. Let $P(n)$ be the number of pencils in n packs, where n is a whole number from 0 through 5. Write $P(n)$ and list its range.

Q39. A tank contains $V(t) = 24 - 3t$ liters after t minutes, for $0 \leq t \leq 8$. Find $V(5)$, and explain why $t = 9$ is outside this model.

Q40. A club charges $C(n) = 15 + 4n$ dollars for a group of 1 to 6 students. If a group pays \$35, how many students are there? Check the input against the domain.

Homework review: R1–R5

R1. A mapping sends $-4 \rightarrow 6$, $0 \rightarrow 6$, and $3 \rightarrow 9$. Is it a function? Explain.

R2. Does the relation $(0, 2)$, $(3, 5)$, $(0, 8)$ define a function? Explain.

R3. The relation is $x = y^2 + 1$. At $x = 5$, find the y -values. Does y define a function of x ?

R4. Find the real-number domain of $f(x) = \frac{3}{x-6}$.

R5. Find the range of $f(x) = x^2 - 1$ for domain $\{-2, 0, 2\}$.

Homework review: R6–R8

R6. A function contains the ordered pair $(-3, 10)$. Write it as a statement about g and identify its input.

R7. For $h(x) = x^2 + 2x - 1$, find $h(-3)$.

R8. A craft session costs \$9 plus \$3 per student, for groups of 1 to 4 students. Write $C(n)$ and list its domain and range.

Homework word problems: W1–W5

W1. A school assigns exactly one locker to each student. Each student is identified by a unique ID. Does the assignment from student ID to locker number define a function? Explain.

W2. During the same school term, each student ID is paired with the names of all that student's classes. One student takes math, art, and science. Is class name a function of student ID in this relation?

W3. A bakery's function $B(n)$ gives the total price in dollars for n muffins. Interpret $B(6) = 18$, including units.

W4. A photo booth charges \$2 per print. A customer may buy 1, 2, 3, or 4 prints. Write $P(n)$ and list its domain and range.

W5. A bowling outing costs \$7 for shoe rental plus \$6 per game. A player may play 1 through 4 whole games. Write $C(g)$, then find and interpret $C(3)$.

Homework word problems: W6–W10

W6. A theater charges a \$10 booking fee plus \$8 per ticket. Orders may contain 1 through 5 whole tickets. Write the cost function, give its domain, and list all possible total costs.

W7. A bicycle rental costs \$5 plus \$4 per hour and is prorated continuously. Rentals last from 0.5 through 3 hours. Write $C(t)$, find $C(1.5)$, and give the domain and range.

W8. A candle's height is $H(t) = 18 - 2t$ centimeters for $0 \leq t \leq 9$ hours. Find $H(4)$, give the range, and explain why $t = 10$ is not allowed.

W9. A club orders badges for \$3 each plus a \$6 shipping fee. An order must contain 1 through 12 whole badges. Its total is \$33. Write the function and find the number ordered, checking the domain.

W10. A square garden has side length s meters, allowed to vary continuously from 2 through 5 meters. Write the area function $A(s)$, find $A(3.5)$, and give its domain and range.

Homework word problems: W11–W12

W11. A team rents a bus for \$120 and splits the cost equally among n riders, where n is a whole number from 4 through 8. Write the per-person cost $P(n)$, list its domain and range, and find the smallest per-person cost.

W12. A school fair offers two booth plans: $A(n) = 12 + 3n$ and $B(n) = 6 + 5n$ dollars, where n is a whole number from 1 through 6 activities. Find the input where the costs match. Which plan is cheaper below and above that input within the domain?