



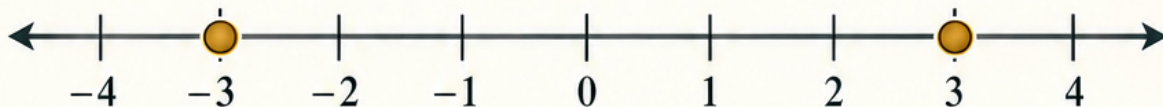
SCHOOL
OF MATH

ALGEBRA 1

CLASS 16

Absolute-Value Equations and Inequalities. Graphing and Solving

$$|x| = 3$$



$$|x| \leq 3$$



Distance • Equations • Inequalities • Applications

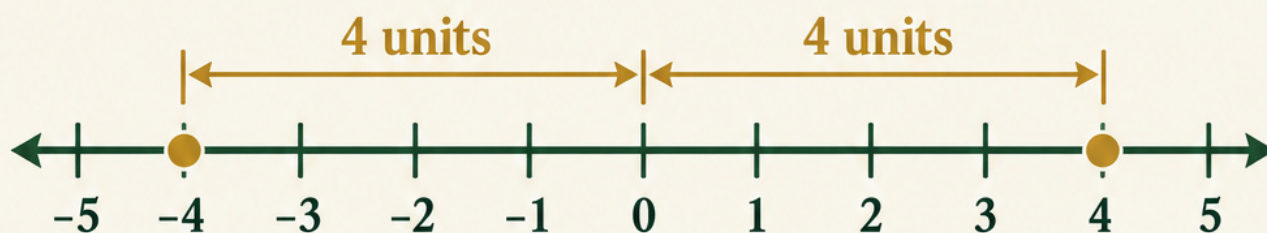
Name: _____

Date: _____

1. Understanding Absolute Value

Distance from zero on a number line

Absolute value tells how far a number is from 0, regardless of direction.



$$|-4| = 4$$

$$|4| = 4$$

Opposite numbers have the same absolute value.

EXAMPLES

$$|-7| = 7$$

$$|3| = 3$$

$$|0| = 0$$



KEY IDEA

Absolute value is always nonnegative.

TRY IT

$$|-9| = \underline{\hspace{2cm}}$$

$$|6| = \underline{\hspace{2cm}}$$

Image 1 practice: Absolute value as distance

Q1. Evaluate $|-13|$ and describe the distance it represents.

Q2. Which is farther from zero, -11 or 8 ? By how many units?

Q3. Write the two numbers exactly 12 units from zero.

Q4. Find the distance between -7 and 5 .

Q5. If $a < 0$, simplify $|a| + a$. Explain without choosing a particular number.

2. Evaluating Absolute-Value Expressions

Positive numbers, negative numbers, and zero.

Evaluate inside the absolute-value bars first.
Then find the distance from zero.

EXAMPLES

$$|8| = 8$$

$$|-8| = 8$$

$$|0| = 0$$

STEP BY STEP

$$\begin{aligned} &|3 - 9| \\ &= |-6| \\ &= 6 \end{aligned}$$

$$\begin{aligned} &2|-5| + 3 \\ &= 2(5) + 3 \\ &= 13 \end{aligned}$$

WATCH THE NEGATIVE SIGN

$$|-4| = 4$$

$$-|4| = -4$$

A negative sign outside the bars stays outside.

TRY IT

$$|-12| = \underline{\hspace{2cm}}$$

$$|7 - 10| = \underline{\hspace{2cm}}$$

$$3|-2| - 1 = \underline{\hspace{2cm}}$$

Image 2 practice: Evaluating expressions

Q6. Evaluate $|8 - 15|$.

Q7. Evaluate $-|-9| + 4$.

Q8. Evaluate $4|2 - 6| - 3$.

Q9. Evaluate $|-3/2| + |1/4 - 1|$.

Q10. Evaluate $|2 - |-7|| - |3 - 8|$.

3. Solving Absolute-Value Equations

Use two cases.

For a positive number a :

$$|A| = a \text{ means } A = a \text{ or } A = -a$$

A number can be the same distance from zero in either direction.

EXAMPLE 1

$$|x| = 5$$

Case 1:

$$x = 5$$

Case 2:

$$x = -5$$

Solutions: $x = -5$ or $x = 5$

EXAMPLE 2

$$|x - 2| = 4$$

Case 1:

$$x - 2 = 4$$

$$x = 6$$

Case 2:

$$x - 2 = -4$$

$$x = -2$$

Solutions: $x = -2$ or $x = 6$

CHECK BOTH SOLUTIONS

$$|6 - 2| = |4| = 4$$

$$|-2 - 2| = |-4| = 4$$

KEY IDEA

Use **OR** to connect the two solutions.

TRY IT

$$|x| = 7 \quad \underline{\hspace{2cm}}$$

$$|x + 3| = 5 \quad \underline{\hspace{2cm}}$$

Image 3 practice: Solving two-case equations

Q11. Solve $|x| = 9$.

Q12. Solve $|x + 5| = 8$.

Q13. Solve $|x - 7| = 3$ and plot the solutions.

Q14. Solve $|2x - 1| = 7$.

Q15. Solve $|3x + 2| = 1$.

4. Isolating the Absolute Value

Isolate first. Then use two cases.

Before splitting into two cases, get the absolute-value expression alone on one side.

STEP 1: ISOLATE

$$3|x - 1| + 2 = 14$$

$$3|x - 1| = 12 \quad \text{Subtract 2 from both sides.}$$

$$|x - 1| = 4 \quad \text{Divide both sides by 3.}$$

STEP 2: USE TWO CASES

Case 1:

$$x - 1 = 4$$

$$x = 5$$

Case 2:

$$x - 1 = -4$$

$$x = -3$$

Solutions: $x = -3$ or $x = 5$

STEP 3: CHECK

$$x = 5: 3|5 - 1| + 2 = 14$$

$$x = -3: 3|-3 - 1| + 2 = 14$$

KEY IDEA

Split into two cases **only after** the absolute value is isolated.

TRY IT

$$2|x + 3| - 5 = 9 \quad \underline{\hspace{2cm}}$$

$$4|x - 2| + 1 = 13 \quad \underline{\hspace{2cm}}$$

Image 4 practice: Isolating before splitting

Q16. Solve $|x - 5| + 4 = 10$.

Q17. Solve $5|x + 1| = 20$.

Q18. Solve $2|x - 4| + 3 = 15$.

Q19. Solve $3|2x + 1| - 4 = 11$.

Q20. Solve $8 - 2|3x - 2| = 0$.

5. No Solution & Checking Answers

Absolute value can never be negative.

After isolating the absolute value, check the other side.

**$|A|$ = positive number:
two cases**

If $|A| = k$ with $k > 0$,
solve $A = k$ or $A = -k$.

**$|A| = 0$:
solve $A = 0$**

If $|A| = 0$,
solve $A = 0$.

**$|A|$ = negative number:
no solution**

If $|A| = k$ with $k < 0$,
there is no solution.

Each case must still be solved and checked.

EXAMPLE 1: NO SOLUTION

$$2|x - 3| + 5 = 1$$

$$2|x - 3| = -4$$

$$|x - 3| = -2$$

No solution.

A distance cannot equal -2 .

EXAMPLE 2: CHECK BOTH ANSWERS

$$|x + 1| = 4$$

$$x = 3 \text{ or } x = -5$$

$$x = 3: |3 + 1| = |4| = 4 \quad \checkmark$$

$$x = -5: |-5 + 1| = |-4| = 4 \quad \checkmark$$

Substitute each answer into the original equation.

WHEN THE RIGHT SIDE IS ZERO

$$|x - 6| = 0 \rightarrow x = 6$$

Zero is the only number whose absolute value is zero.

TRY IT

$$|x + 2| = -7 \quad \underline{\hspace{2cm}}$$

$$|x - 4| = 0 \quad \underline{\hspace{2cm}}$$

Image 5 practice: No solution, one solution, and checking

Q21. Solve $|x - 9| = -4$.

Q22. Solve $|2x + 8| = 0$.

Q23. Solve $4|x - 2| + 6 = 2$.

Q24. Solve $-3|x + 4| = -15$.

Q25. A student says $|x - 3| = 8$ has solutions 11 and -11 . Check both and correct the answer.

6. Absolute-Value Inequalities: $<$ and $\leq$

Less than means **BETWEEN**.

FOR $A > 0$:

$$|x| < a \iff -a < x < a$$

$$|x| \leq a \iff -a \leq x \leq a$$

The solution must satisfy **BOTH** bounds: use **AND**.

EXAMPLE 1: STRICT INEQUALITY

$$|x| < 3$$

$$-3 < x < 3$$



Open circles: endpoints are excluded.

EXAMPLE 2: INCLUSIVE INEQUALITY

$$|x - 1| \leq 2$$

$$-2 \leq x - 1 \leq 2$$

$$-1 \leq x \leq 3$$



Closed circles: endpoints are included.

KEY IDEA

Isolate the absolute value first, then write a compound inequality.

TRY IT

$$|x| \leq 5 \quad \underline{\hspace{2cm}}$$

$$|x + 2| < 3 \quad \underline{\hspace{2cm}}$$

Image 6 practice: Less than: between the bounds

Q26. Solve and graph $|x| < 6$.

Q27. Solve and graph $|x - 4| \leq 3$.

Q28. Solve and graph $|2x + 3| < 7$.

Q29. Solve and graph $3|x + 1| - 2 \leq 10$.

Q30. Solve and graph $|5 - 2x| < 3$.

7. Absolute-Value Inequalities

> and ≥

Greater than means OUTSIDE.

FOR $a > 0$:

$$|x| > a \iff x < -a \text{ or } x > a$$

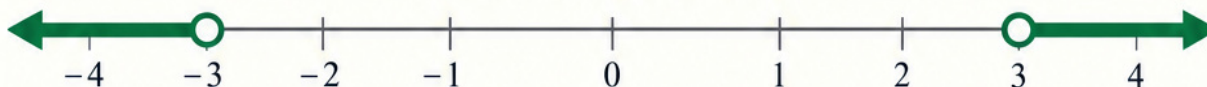
$$|x| \geq a \iff x \leq -a \text{ or } x \geq a$$

Use **OR**: either outer region is a solution.

EXAMPLE 1: STRICT INEQUALITY

$$|x| > 3$$

$$x < -3 \text{ or } x > 3$$



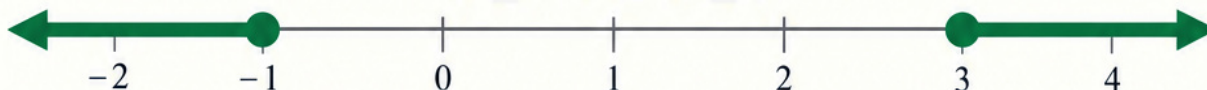
Open circles: endpoints are excluded.

EXAMPLE 2: INCLUSIVE INEQUALITY

$$|x - 1| \geq 2$$

$$x - 1 \leq -2 \text{ or } x - 1 \geq 2$$

$$x \leq -1 \text{ or } x \geq 3$$



Closed circles: endpoints are included.

KEY IDEA

Greater than: shade **away from the center**.

TRY IT

$$|x| \geq 5 \quad \underline{\hspace{2cm}} \quad |x + 2| > 3 \quad \underline{\hspace{2cm}}$$

Image 7 practice: Greater than: outside the bounds

Q31. Solve and graph $|x| > 8$.

Q32. Solve and graph $|x + 4| \geq 2$.

Q33. Solve and graph $|3x - 6| > 9$.

Q34. Solve and graph $2|x - 2| + 1 \geq 11$.

Q35. Solve each: (a) $|x + 6| < 0$; (b) $|x + 6| \geq 0$; (c) $|x + 6| > 0$.

8. Real-World Applications

Distance and tolerance.

KEY CONCEPT

$|x - c|$ measures the distance between x and c .

c = reference value; x = actual value

DISTANCE: EXACTLY HOW FAR?

A cyclist is exactly 3 miles from a checkpoint at mile 10. At which mile markers could the cyclist be?

$$|x - 10| = 3$$

$$x - 10 = 3 \text{ or } x - 10 = -3$$

$$x = 13 \text{ or } x = 7$$

The cyclist could be at mile 7 or mile 13.



TOLERANCE: AN ACCEPTABLE RANGE

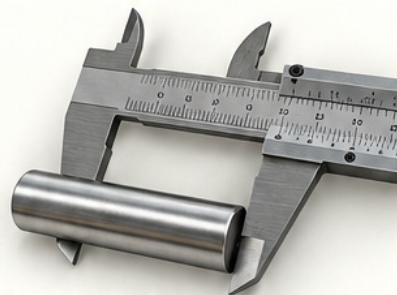
A machine makes rods with a target length of 20 cm. The length may differ from the target by at most 0.2 cm.

$$|L - 20| \leq 0.2$$

$$-0.2 \leq L - 20 \leq 0.2$$

$$19.8 \leq L \leq 20.2$$

Acceptable lengths: 19.8 cm through 20.2 cm, including both endpoints.



KEY IDEA

Exactly a units away:

$$|x - c| = a$$

Within a units:

$$|x - c| \leq a$$

TRY IT

A refrigerator should stay within 2°F of 38°F .

Write and solve an absolute-value inequality for temperature T .

Image 8 practice: Distance and tolerance

Q36. A hiker is exactly 4 miles from mile marker 18. Write an equation and find both possible mile markers.

Q37. A thermostat should remain within 3°C of 21°C , including the boundary temperatures. Write and solve an inequality.

Q38. A package should weigh 500 g, with an allowed difference of at most 8 g. Is a 491 g package acceptable? Give the full accepted range.

Q39. A sensor sounds an alarm when temperature differs from 30°C by more than 5°C . Write and solve its alarm condition.

Q40. A part is accepted when its length is between 7.85 cm and 8.15 cm, inclusive. Express the rule as one absolute-value inequality.

Homework review: R1–R5

R1. Find the distance between -9 and 4 , and explain why it is not negative.

R2. Evaluate $2|5 - 12| - |-6|$.

R3. Solve $|x + 2| = 9$.

R4. Solve $4|2x - 3| + 1 = 21$.

R5. Solve $7 - |x - 1| = 9$ and explain the result.

Homework review: R6–R8

R6. Solve and graph $|2x - 4| \leq 6$.

R7. Solve and graph $|2x + 4| > 8$.

R8. A bottle targets 250 mL with a tolerance of 4 mL, including both limits. Write and solve the rule.

Homework word problems: W1–W5

W1. A submarine is at -16 m relative to sea level. How far is it from sea level?

W2. One floor is labeled -3 and another is labeled 8 . How many floor intervals separate them?

W3. A treasure is exactly 6 m from a flag at position 15 m on a straight path. Find both positions.

W4. A game awards 3 points for each unit a marker is from position 4 . A player earns 18 points. Find the marker positions.

W5. A room must stay within 2.5°C of 20°C , including the limits. Write the acceptable temperature interval.

Homework word problems: W6–W10

W6. A balance accepts packets from 98 g through 102 g, inclusive. Write an absolute-value inequality and decide whether 102.1 g passes.

W7. A runner must be less than 4 km from a station at kilometer 12. Write and graph the allowed positions x .

W8. An alarm activates when pressure is at least 7 kPa away from 50 kPa. Write and solve the condition for nonnegative pressure p .

W9. A robot follows $x = 2t + 1$ meters, with time $t \geq 0$ seconds. When is it exactly 5 m from position 9 m?

W10. A sensor reading is $R = 3t - 2$ units for $t \geq 0$ seconds. The safe reading differs from 10 by at most 6 units. Find the safe time interval.

Homework word problems: W11–W12

W11. A rod must satisfy both $|L - 25| \leq 0.4$ and $|L - 25.2| \leq 0.3$, with lengths in centimeters. Find all accepted lengths.

W12. A drone moves along a straight track with $x = 4t - 3$ meters for $0 \leq t \leq 8$ seconds. It is in a restricted zone when it is less than 5 m from position 13 m. Find the times outside the restricted zone, including its boundary.