

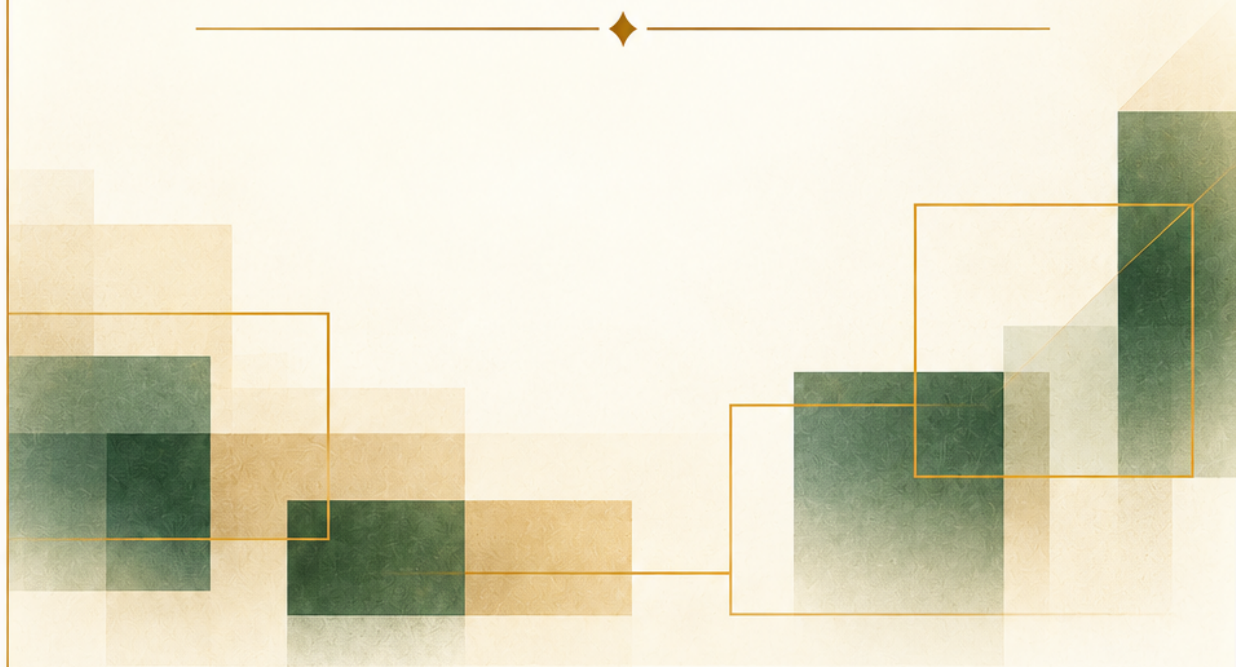


ALGEBRA

CLASS 17

Factoring Trinomials

Leading coefficient = 1 and $\neq 1$



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1. Understanding Trinomial Structure

The first step to factoring

$$ax^2 + bx + c$$

a = leading
coefficient

b = coefficient
of x

c = constant

$$a \neq 0$$

A trinomial has three terms.
In a quadratic trinomial, the highest power is 2.

Leading coefficient = 1

$$x^2 + 5x + 6$$

$$a = 1, \quad b = 5, \quad c = 6$$

Leading coefficient $\neq 1$

$$2x^2 + 7x + 3$$

$$a = 2, \quad b = 7, \quad c = 3$$

Factoring reverses multiplication

$$(x + 2)(x + 3) \xrightleftharpoons[\text{Factor}]{\text{Multiply}} x^2 + 5x + 6$$

Check: $x^2 + 3x + 2x + 6 = x^2 + 5x + 6$



Try it: Identify a , b , and c in $3x^2 - 8x + 4$.

Understanding trinomial structure

Q1. Identify a, b, and c, then factor.

$$x^2 + 8x + 15$$

Q2. Identify a, b, and c. State whether a equals 1, then factor.

$$2x^2 + 5x + 2$$

Q3. Identify the signed coefficients, then factor.

$$4x^2 - 9x + 2$$

Q4. Rewrite in standard form, identify a, b, and c, then factor.

$$24 + 11x + x^2$$

Q5. Identify the original signed coefficients. Factor out a negative factor before completing the factorization.

$$-3x^2 + 10x - 8$$

2. Check for a Greatest Common Factor

Always look for a GCF first

The **GCF** is the largest factor shared by every term.
Factor it out before factoring the trinomial.

Example: Factor completely

$$3x^2 + 15x + 18$$

- 1 Find the GCF.
GCF of 3, 15, and 18 = 3
No x is common to all three terms.

- 2 Divide every term by 3.
$$3x^2 + 15x + 18 = 3(x^2 + 5x + 6)$$

- 3 Factor the remaining trinomial.

$$2 \times 3 = 6$$

$$2 + 3 = 5$$

$$3(x + 2)(x + 3)$$

Variables can be part of the GCF

$$\begin{aligned} &2x^3 + 10x^2 + 12x \\ &= 2x(x^2 + 5x + 6) \\ &= 2x(x + 2)(x + 3) \end{aligned}$$

Remember: | Keep the GCF outside every step.



Try it: Factor completely.

$$4x^2 + 20x + 24$$

Check the greatest common factor first

Q6. Factor completely over the integers.

$$4x^2 + 28x + 40$$

Q7. Factor completely over the integers.

$$6x^2 - 42x + 60$$

Q8. Factor completely over the integers.

$$3x^3 + 24x^2 + 45x$$

Q9. Factor completely over the integers.

$$-2x^2 - 18x - 40$$

Q10. Factor completely over the integers.

$$12x^4 - 30x^3 + 12x^2$$

3. Factoring When $a = 1$

Positive constant: $c > 0$

If $a = 1$, then $x^2 + bx + c = (x + m)(x + n)$.

Find two integers: $mn = c$ and $m + n = b$.

When $c > 0$, the two numbers have the same sign.

Positive middle term → both positive

Example:

$$x^2 + 7x + 12$$

Pair	Product	Sum
1 and 12	12	13
2 and 6	12	8
3 and 4	12	7

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

Negative middle term → both negative

Example:

$$x^2 - 7x + 12$$

$$(-3)(-4) = 12$$

$$(-3) + (-4) = -7$$

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

Check by multiplying:

$$(x - 3)(x - 4) = x^2 - 4x - 3x + 12 = x^2 - 7x + 12$$



Try it: Factor each trinomial.

1.

$$x^2 + 9x + 20$$

2.

$$x^2 - 9x + 20$$

Leading coefficient 1: positive constant

Q11. Factor completely over the integers.

$$x^2 + 10x + 21$$

Q12. Factor completely over the integers.

$$x^2 + 12x + 35$$

Q13. Factor completely over the integers.

$$x^2 - 11x + 28$$

Q14. Factor completely over the integers.

$$x^2 - 17x + 72$$

Q15. Factor completely over the integers.

$$x^2 - 23x + 126$$



4. Factoring When $a = 1$

Negative constant: $c < 0$

If $a = 1$, then $x^2 + bx + c = (x + m)(x + n)$.

Find two integers: $mn = c$ and $m + n = b$.

When $c < 0$, the two numbers have opposite signs.

Positive middle term → both opposite signs

Example:

$$x^2 + 2x - 15$$

Use 5 and -3 :

$$5 \times (-3) = -15$$

$$5 + (-3) = 2$$

$$x^2 + 2x - 15 = (x + 5)(x - 3)$$

Negative middle term → both opposite signs

Example:

$$x^2 - 2x - 15$$

Use -5 and 3:

$$(-5) \times 3 = -15$$

$$-5 + 3 = -2$$

$$x^2 - 2x - 15 = (x - 5)(x + 3)$$

The number with the greater absolute value has the same sign as b .

Check by multiplying:

$$\begin{aligned}(x + 5)(x - 3) &= x^2 - 3x + 5x - 15 \\ &= x^2 + 2x - 15\end{aligned}$$



Try it: Factor each trinomial.

1.

$$x^2 + 3x - 18$$

2.

$$x^2 - 3x - 18$$

Leading coefficient 1: negative constant

Q16. Factor completely over the integers.

$$x^2 + 5x - 24$$

Q17. Factor completely over the integers.

$$x^2 - 5x - 36$$

Q18. Factor completely over the integers.

$$x^2 + 2x - 63$$

Q19. Factor completely over the integers.

$$x^2 - 4x - 77$$

Q20. Factor completely over the integers.

$$x^2 - 3x - 154$$

5. The *ac* Method

Factoring when $a \neq 1$

For $ax^2 + bx + c$,
find two integers whose:

Product = ac

Sum = b

Check for a GCF first.

Example: $6x^2 + 11x + 3$

1. Identify the coefficients.

$$a = 6, b = 11, c = 3$$

2. Multiply a and c .

$$ac = 6 \times 3 = 18$$

3. Find a pair with product **18** and sum **11**.

Pair	Product	Sum
1 and 18	18	19
2 and 9	18	11
3 and 6	18	9

4. Split the middle term.

$$6x^2 + 11x + 3 = 6x^2 + 2x + 9x + 3$$

$2x + 9x = 11x$, so the expression is unchanged.

Finish by grouping.

$$\begin{aligned} 6x^2 + 2x + 9x + 3 &= 2x(3x + 1) + 3(3x + 1) \\ &= (3x + 1)(2x + 3) \end{aligned}$$

*The pair 2 and 9 splits the middle term;
it does not directly give the binomial constants.*



Try it: For $6x^2 + 7x + 2$, find ac and the pair that adds to b .

The ac method for a leading coefficient other than 1

Q21. Factor completely over the integers.

$$2x^2 + 9x + 4$$

Q22. Factor completely over the integers.

$$3x^2 + 11x + 6$$

Q23. Factor completely over the integers.

$$4x^2 + 11x + 6$$

Q24. Factor completely over the integers.

$$5x^2 + 17x + 6$$

Q25. Factor completely over the integers.

$$6x^2 + 19x + 10$$



6. Split and Group

Turn three terms into four

Split bx into two terms whose coefficients multiply to ac and add to b .

Group in pairs and look for a *common binomial*.

Example: $6x^2 + x - 2$

1. Find the pair.

$$ac = 6(-2) = -12$$

$$4 \times (-3) = -12$$

$$4 + (-3) = 1$$

2. Split the middle term.

$$\begin{aligned} 6x^2 + x - 2 \\ = 6x^2 + 4x - 3x - 2 \end{aligned}$$

3. Group and factor each pair.

$$\begin{aligned} (6x^2 + 4x) + (-3x - 2) \\ = 2x(3x + 2) - 1(3x + 2) \end{aligned}$$

4. Factor out the common binomial.

$$(3x + 2)(2x - 1)$$

Watch the negative sign.

$$-3x - 2 = -1(3x + 2)$$

Factoring out -1 changes both signs inside.

Check by multiplying.

$$\begin{aligned} (3x + 2)(2x - 1) \\ = 6x^2 - 3x + 4x - 2 \\ = 6x^2 + x - 2 \end{aligned}$$



Try it: Factor $6x^2 - x - 2$ by grouping.

Split and group with signed terms

Q26. Factor completely over the integers.

$$2x^2 + 7x - 4$$

Q27. Factor completely over the integers.

$$3x^2 - 10x + 3$$

Q28. Factor completely over the integers.

$$4x^2 + 4x - 15$$

Q29. Factor completely over the integers.

$$6x^2 - 13x + 6$$

Q30. Factor completely over the integers.

$$8x^2 - 2x - 3$$

7. Special Cases

Perfect squares and integer factoring

Recognize perfect-square trinomials.

$$u^2 + 2uv + v^2 = (u + v)^2$$

$$u^2 - 2uv + v^2 = (u - v)^2$$

The first and last terms are *squares*. The middle term is $\pm 2uv$.

1. *Example:*

$$x^2 + 10x + 25 = (x + 5)^2$$

$$2(x)(5) = 10x$$

2. *Example:*

$$4x^2 - 12x + 9 = (2x - 3)^2$$

$$-2(2x)(3) = -12x$$

Square end terms alone are *not* enough: check the middle term.

Not every trinomial factors over the integers.

$$x^2 + x - 1$$

Need: $mn = -1$ and $m + n = 1$.

The only integer pair is 1 and -1.

$$1 + (-1) = 0, \text{ not } 1.$$

Therefore, $x^2 + x - 1$ is *irreducible* over the integers.

It can still be factored using irrational numbers.



Try it: Factor over the integers, or state irreducible.

1. $x^2 - 14x + 49$

2. $x^2 + 3x + 1$

Perfect squares and integer-irreducible trinomials

Q31. Factor completely over the integers.

$$x^2 + 12x + 36$$

Q32. Factor completely over the integers.

$$9x^2 - 30x + 25$$

Q33. Factor completely over the integers.

$$3x^2 - 24x + 48$$

Q34. Factor over the integers, or explain why no integer factorization exists.

$$x^2 + x - 4$$

Q35. Test all relevant ac pairs and factor over the integers, or state irreducible.

$$2x^2 + 3x + 3$$

8. Check and Apply Your Factors

From factoring to solving equations

Check by multiplying.

$$\begin{aligned}(2x - 1)(x + 3) \\ &= 2x^2 + \color{brown}{6}x - \color{brown}{x} - 3 \\ &= 2x^2 + \color{brown}{5}x - 3\end{aligned}$$

The expanded expression *must match* the original trinomial.

Zero-product property.

If $\color{brown}{A}B = 0$, then $\color{brown}{A} = 0$ or $\color{brown}{B} = 0$.

Set the equation equal to zero before using this property.

Solve: $2x^2 + 5x = 3$.

1. Set equal to zero.

$$2x^2 + 5x - 3 = 0$$

2. Factor.

$$(2x - 1)(x + 3) = 0$$

3. Set each factor equal to zero.

$2x - 1 = 0$	$x + 3 = 0$
$x = \frac{1}{2}$	$x = -3$

4. Check in the original equation.

$$\begin{aligned}x = \frac{1}{2}: \quad & 2\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) = 3 \\ x = -3: \quad & 2(-3)^2 + 5(-3) = 3\end{aligned}$$

Solutions: $x = \frac{1}{2}$ or $x = -3$.

Factoring rewrites an expression. *Solving* finds values of x .



Try it: Solve by factoring.

1. $x^2 - 5x + 6 = 0$

2. $3x^2 + 2x - 1 = 0$

Check and apply your factors

Q36. A student claims the factors are $(3x + 4)(x + 2)$. Expand that proposal, explain the error, and factor correctly.

$$3x^2 + 14x + 8$$

Q37. Solve by factoring and check both solutions.

$$x^2 - 8x + 15 = 0$$

Q38. Solve by factoring. Put the equation in zero-product form first.

$$2x^2 + 7x = 15$$

Q39. Solve by factoring and retain exact fractional solutions.

$$3x^2 - 14x = -8$$

Q40. Solve by factoring. Explain why dividing by x at the start could lose a solution.

$$2x^3 - 14x^2 + 20x = 0$$

Homework review

R1. Identify a, b, and c, then factor completely.

$$5x^2 - 19x + 12$$

R2. Factor completely over the integers.

$$6x^2 + 54x + 120$$

R3. Factor completely over the integers.

$$x^2 - 16x + 63$$

R4. Factor completely over the integers.

$$x^2 + 4x - 45$$

Homework review

R5. Use the ac method and show the matching pair.

$$3x^2 + 13x + 4$$

R6. Split the middle term and factor by grouping.

$$10x^2 - x - 3$$

R7. Factor over the integers, or justify why it is irreducible.

$$x^2 + 4x + 7$$

R8. Solve by factoring and check the roots.

$$4x^2 - 4x = 15$$

Homework word problems

W1. A rectangle has area $x^2 + 9x + 18$ square centimeters and one side $x + 3$ cm, with x positive. Factor the area to find the other side. Then find both side lengths and the area when $x = 4$.

W2. A rectangular tile has area $x^2 - 10x + 21$ square units. One side is $x - 3$ units and $x > 7$. Find the other side by factoring, then evaluate both sides and the area at $x = 10$.

W3. A rectangular mat has area $2x^2 + 11x + 12$ square feet. Its width is $2x + 3$ feet and x is positive. Factor to find its length. Evaluate the dimensions and area at $x = 2$.

W4. A square display has area $x^2 + 18x + 81$ square centimeters, where x is nonnegative. Find a formula for its side length, then find the perimeter when $x = 3$.

Homework word problems

W5. Three identical rectangular banners have total area $3x^2 + 21x + 30$ square feet. Each has width $x + 2$ feet and x is positive. Factor the total area to find each banner's length. Find the area of one banner and the total area at $x = 4$.

W6. A rectangular garden is 5 meters longer than it is wide and has area 84 square meters. Use factoring to find its dimensions.

W7. The product of two consecutive positive integers is 132. Model the situation with a quadratic equation and find the integers.

W8. A rectangular sign is 3 centimeters longer than twice its width. Its area is 65 square centimeters. Find its width and length by factoring.

Homework word problems

W9. A 6-meter by 10-meter garden is surrounded on all four sides by a path of uniform width x meters. The garden and path together cover 96 square meters. Find the path width.

W10. A fundraiser sells x gift bags at a price of $30 - x$ dollars per bag. Find the positive whole-number values of x that give revenue \$200, with a positive price. State the price for each option.

W11. A model gives a ball's height as $h(x) = -5x^2 + 15x + 20$ meters, where x is elapsed time in seconds. According to this model, when does the ball reach the ground after release?

W12. An open box is made by cutting squares of side x cm from each corner of an 18 cm by 26 cm sheet, then folding up the sides. The base area must be 240 cm^2 . Find the physically valid cut size and the resulting box volume.