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ALGEBRA 1 • CLASS 18

Solving Quadratics by Factoring

Turn equations into factors. Find the solutions.

$$x^2 + bx + c = 0$$

$$(x + m)(x + n) = 0$$

$$m + n = b \quad \bullet \quad mn = c$$

FACTOR • SOLVE • CHECK

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Quadratic Equations & Standard Form

What is a quadratic equation?

An equation whose highest power of x is 2.

$$ax^2 + bx + c = 0$$

$$a \neq 0$$

a: coefficient of x^2	b: coefficient of x	c: constant term

Identify the coefficients

$$2x^2 + 7x + 3 = 0$$

$a = 2$ $b = 7$ $c = 3$

Write in standard form

$$x^2 + 5x = 14$$

↓ Subtract 14 from both sides

$$x^2 + 5x - 14 = 0$$

Before factoring, set the equation equal to zero.

Quadratic equations and standard form

Q1. Identify a, b, and c in the quadratic equation.

$$3x^2 - 8x + 5 = 0$$

Q2. Identify a, b, and c. Which term is missing?

$$x^2 - 9x = 0$$

Q3. Write the equation in standard form, then identify a, b, and c.

$$x^2 + 6x = 27$$

Q4. Write the equation in standard form, then identify a, b, and c.

$$2x^2 = 5x + 12$$

Q5. Decide whether each equation is quadratic: 4x − 7 = 0 and (x + 1)(x − 2) = 0. Explain.

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The Zero-Product Property

If two factors multiply to zero, at least one factor must be zero.

$$AB = 0$$

$$A = 0 \quad \text{or} \quad B = 0$$

Example

$$0 \times 7 = 0$$

Example

$$5 \times 0 = 0$$

Solve a factored equation

$$(x - 3)(x + 2) = 0$$

$$x - 3 = 0 \rightarrow x = 3 \quad \text{OR} \quad x + 2 = 0 \rightarrow x = -2$$

Solutions:

$$x = 3 \quad \text{or} \quad x = -2$$

Check both solutions

$x = 3$:

$$(3 - 3)(3 + 2) = 0 \times 5 = 0$$

$x = -2$:

$$(-2 - 3)(-2 + 2) = (-5) \times 0 = 0$$

Set each factor equal to zero, then solve.

Use this property only when the product equals zero.

The zero-product property

Q6. Solve the equation.

$$(x - 6)(x + 4) = 0$$

Q7. Solve the equation.

$$x(x + 9) = 0$$

Q8. Solve the equation.

$$(2x - 5)(x + 1) = 0$$

Q9. Solve the equation.

$$(3x + 2)(4x - 1) = 0$$

Q10. A student writes x − 2 = 8 or x + 5 = 8 for this equation. Explain the error, then solve correctly.

$$(x - 2)(x + 5) = 8$$

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Factoring Out the Greatest Common Factor

Look for a factor shared by every term.

$$ax + bx = x(a + b)$$

Worked Example

Solve: $x^2 - 5x = 0$

1 **Identify the GCF.**

Both terms contain x.
GCF = x.

2 **Factor.**

$$x(x - 5) = 0$$

3 **Use the zero-product property.**

$x = 0$ OR $x - 5 = 0 \Rightarrow x = 5$

Solutions:
$x = 0$ or $x = 5$

Example with a Numerical GCF

Solve: $3x^2 + 12x = 0$

1 **Identify the GCF.**

Both terms contain $3x$.
GCF = $3x$.

2 **Factor.**

$$3x(x + 4) = 0$$

3 **Use the zero-product property.**

$3x = 0 \Rightarrow x = 0$ OR $x + 4 = 0 \Rightarrow x = -4$

Solutions:
$x = 0$ or $x = -4$

Do not divide both sides by x:
you could lose the solution $x = 0$.

Factoring out the greatest common factor

Q11. Solve by factoring out the greatest common factor.

$$x^2 - 7x = 0$$

Q12. Solve by factoring out the greatest common factor.

$$4x^2 + 20x = 0$$

Q13. Solve. Begin by writing the equation in standard form.

$$6x^2 = 9x$$

Q14. A student divides both sides by x and reports the single answer x = 2. Find all solutions and explain what the student missed.

$$5x^2 - 10x = 0$$

Q15. Solve by factoring out the greatest common factor, including the negative sign.

$$-3x^2 + 18x = 0$$

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Factoring Trinomials
Leading Coefficient = 1

For $x^2 + bx + c$, find two numbers that:

Multiply to c	Add to b

$$x^2 + (m+n)x + mn = (x+m)(x+n)$$

Example 1: Positive Constant
Solve: $x^2 + 7x + 12 = 0$

1. **Find the pair.**
$$3 \times 4 = 12$$
$$3 + 4 = 7$$
2. **Factor.**
$$(x+3)(x+4) = 0$$
3. **Set each factor equal to zero.**
$$x+3=0 \quad \text{OR} \quad x+4=0$$

Solution:
$$x = -3 \text{ or } x = -4$$

Example 2: Negative Constant
Solve: $x^2 - x - 12 = 0$

1. **Find the pair.**
$$(-4) \times 3 = -12$$
$$-4 + 3 = -1$$
2. **Factor.**
$$(x-4)(x+3) = 0$$
3. **Set each factor equal to zero.**
$$x-4=0 \quad \text{OR} \quad x+3=0$$

Solution:
$$x = 4 \text{ or } x = -3$$

⚠ **Factor first. Then use the zero-product property.**

Factoring trinomials with leading coefficient 1

Q16. Solve by factoring.

$$x^2 + 9x + 20 = 0$$

Q17. Solve by factoring.

$$x^2 - 8x + 15 = 0$$

Q18. Solve by factoring.

$$x^2 + 2x - 35 = 0$$

Q19. Solve by factoring.

$$x^2 - 3x - 28 = 0$$

Q20. Solve by factoring.

$$x^2 - 11x + 24 = 0$$

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Factoring Trinomials
Leading Coefficient ≠ 1

For $ax^2 + bx + c = 0$, find two numbers that multiply to ac and add to b.

Solve: $2x^2 + 7x + 3 = 0$

1 Multiply a and c.

$$2 \times 3 = 6$$

2 Find the pair.

$$6 \times 1 = 6$$
$$6 + 1 = 7$$

3 Split the middle term and group.

$$2x^2 + 6x + x + 3 = 0$$
$$2x(x + 3) + 1(x + 3) = 0$$
$$(2x + 1)(x + 3) = 0$$

4 Use the zero-product property.

$$2x + 1 = 0 \quad x = -\frac{1}{2}$$

OR

$$x + 3 = 0 \quad x = -3$$

Solutions: $x = -\frac{1}{2}$ or $x = -3$

Multiply → Split → Group → Solve.
Always check for a greatest common factor first.

Factoring trinomials with the ac method

Q21. Solve using the ac method.

$$2x^2 + 9x + 4 = 0$$

Q22. Solve using the ac method.

$$3x^2 - 10x + 8 = 0$$

Q23. Solve using the ac method.

$$5x^2 + 13x - 6 = 0$$

Q24. Solve using the ac method.

$$4x^2 - 4x - 15 = 0$$

Q25. Solve using the ac method.

$$6x^2 + 7x - 3 = 0$$

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Special Factoring Patterns
Solving Quadratic Equations

Difference of Squares

$$a^2 - b^2 = (a - b)(a + b)$$

Solve: $x^2 - 25 = 0$

1 Recognize two squares.

$$x^2 - 5^2 = 0$$

2 Factor.

$$(x - 5)(x + 5) = 0$$

3 Set each factor equal to zero.

$$x - 5 = 0 \quad \text{or} \quad x + 5 = 0$$

Solutions:

$$x = 5 \quad \text{or} \quad x = -5$$

Perfect-Square Trinomial

$$a^2 - 2ab + b^2 = (a - b)^2$$

Solve: $x^2 - 6x + 9 = 0$

1 Recognize the pattern.

$$x^2 - 2(x)(3) + 3^2 = 0$$

2 Factor.

$$(x - 3)^2 = 0$$

3 Set the repeated factor equal to zero.

$$x - 3 = 0$$

Solution:

$$x = 3$$

One distinct solution (a double root).

Recognize the pattern → Factor → Set factors equal to zero.

Special factoring patterns

Q26. Solve using a special factoring pattern.

$$x^2 - 49 = 0$$

Q27. Solve using a special factoring pattern. How many distinct solutions are there?

$$x^2 + 10x + 25 = 0$$

Q28. Solve using a special factoring pattern.

$$4x^2 - 9 = 0$$

Q29. Solve using a special factoring pattern.

$$9x^2 - 12x + 4 = 0$$

Q30. Solve. Look for a greatest common factor before using a pattern.

$$3x^2 - 48 = 0$$

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Rearranging & Checking Solutions

Set the equation equal to zero before factoring.

Solve: $x^2 + 5x = 14$

1 **Rearrange into standard form.**

Subtract 14 from both sides.

$$x^2 + 5x - 14 = 0$$

2 **Factor.**

Find two numbers that multiply to -14 and add to 5: 7 and -2.

$$(x + 7)(x - 2) = 0$$

3 **Solve each factor.**

$x + 7 = 0$
$x = -7$

or

$x - 2 = 0$
$x = 2$

Solutions:

$x = -7$ or $x = 2$

Check in the ORIGINAL equation: $x^2 + 5x = 14$.

$x = -7$

$(-7)^2 + 5(-7)$
$= 49 - 35 = 14$ ✓

$x = 2$

$2^2 + 5(2)$
$= 4 + 10 = 14$ ✓

Rearrange → Factor → Solve → Check.

Rearranging and checking solutions

Q31. Solve. Check both solutions in the original equation.

$$x^2 - 4x = 21$$

Q32. Solve. Check both solutions in the original equation.

$$x^2 = 6x - 8$$

Q33. Solve. Check both solutions in the original equation.

$$2x^2 + 5x = 3$$

Q34. Solve. Expand the left side first.

$$x(x + 2) = 24$$

Q35. Solve. Expand the left side first.

$$(x - 1)(x + 3) = 12$$

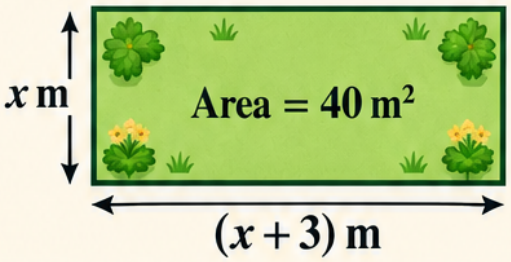
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Quadratic Word Problems

Model • Factor • Solve • Interpret

A rectangular garden has an area of 40 m². Its length is 3 m greater than its width. Find its dimensions.



1 **Define and model.**

Let x be the width, so $x > 0$.

$$x(x+3) = 40$$

2 **Write in standard form.**

$$x^2 + 3x - 40 = 0$$

3 **Factor and solve.**

$$(x+8)(x-5) = 0$$

$$x = -8 \quad \text{or} \quad x = 5$$

4 **Interpret the solutions.**

Reject $x = -8$: a width cannot be negative.

Width = 5 m

Length = 5 + 3 = 8 m

Final Answer: The garden is 5 m wide and 8 m long.

Check: $5 \times 8 = 40$ m² ✓

A solution must make sense in the original situation.

Quadratic word problems

Q36. A rectangle has an area of 84 m^2. Its length is 5 m more than its width. Find the dimensions.

Q37. The product of two consecutive positive integers is 56. Find the integers.

Q38. A ball is kicked from the ground. Its height in feet after t seconds is $h = -16t^2 + 48t$. When does the ball hit the ground?

Q39. Each side of a square is increased by 4 cm, and the new square has an area of 100 cm^2. Find the side length of the original square.

Q40. A triangle has an area of 30 cm^2. Its height is 4 cm more than its base. Find the base and the height.

Homework review

R1. Write the equation in standard form and identify a, b, and c.

$$3x^2 = 7x - 2$$

R2. Solve the equation.

$$(x + 8)(2x - 3) = 0$$

R3. Solve by factoring out the greatest common factor.

$$10x^2 + 15x = 0$$

R4. Solve by factoring.

$$x^2 - 5x - 36 = 0$$

R5. Solve using the ac method.

$$3x^2 + 11x + 6 = 0$$

Homework review (continued)

R6. Solve using a special factoring pattern.

$$16x^2 - 25 = 0$$

R7. Solve. Check both solutions in the original equation.

$$x^2 + 7x = -10$$

R8. A rectangular poster has an area of 70 square inches. Its length is 3 inches more than its width. Find the dimensions.

Homework word problems

W1. A rectangular patio has an area of 60 m². Its length is 4 m more than its width. Find the dimensions.

W2. The product of two consecutive positive odd integers is 99. Find the integers.

W3. The square of a number is 5 more than 4 times the number. Find all possible numbers.

W4. A ball is thrown upward from the ground. Its height in feet after t seconds is $h = -16t^2 + 64t$. When does the ball land?

W5. A model rocket's height in meters after t seconds is $h = -5t^2 + 30t$. At what times is the rocket 40 m high?

Homework word problems (continued)

W6. An 8-inch by 10-inch photo is surrounded by a frame of uniform width. The total area of the photo and frame is 143 square inches. How wide is the frame?

W7. A triangle has an area of 36 cm². Its base is 6 cm more than its height. Find the base and the height.

W8. The sum of a number and its square is 72. Find all possible numbers.

W9. A rectangular pool is 10 m by 16 m. A walkway of uniform width surrounds it, and the pool and walkway together cover 280 m². How wide is the walkway?

W10. A club sells T-shirts. If the price is p dollars, it sells 40 − p shirts, so the revenue is p(40 − p) dollars. What prices give a revenue of exactly $300?

Homework word problems (continued)

W11. A stone is thrown upward from a 48-foot cliff. Its height in feet after t seconds is $h = -16t^2 + 32t + 48$. When does the stone reach the ground?

W12. The legs of a right triangle differ by 7 cm, and the hypotenuse is 13 cm. Find the lengths of the legs.