SCHOOL OF MATH

ALGEBRA 1 — CLASS 20

Solving Quadratics

The Quadratic Formula & the Discriminant

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = b^2 - 4ac$$

$\Delta > 0$:	$\Delta = 0$:	$\Delta < 0$:
Two real solutions	*One real solution*	*No real solutions*

Solve • Predict • Interpret

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 1

Standard Form & Coefficients

Prepare your equation for the quadratic formula.

$$ax^2 + bx + c = 0 \quad | \quad a \neq 0$$

a | coefficient of x^2	b | coefficient of x	c | constant term

1. Already in standard form

$$2x^2 - 5x + 3 = 0$$

$a = 2 \quad b = -5 \quad c = 3$

2. Rearrange first

$$3x^2 + 7 = 4x$$
$$3x^2 - 4x + 7 = 0$$

$a = 3 \quad b = -4 \quad c = 7$

3. Watch for missing terms

$$x^2 - 9 = 0$$
$$x^2 + 0x - 9 = 0$$

$a = 1 \quad b = 0 \quad c = -9$

Remember: Keep the sign with each coefficient.

Standard form and coefficients

Q1. Identify a, b, and c.

$$4x^2 + 9x - 2 = 0$$

Q2. Write the equation in standard form. Then identify a, b, and c.

$$5x^2 = 3x + 8$$

Q3. Identify a, b, and c. Watch for a missing term.

$$x^2 + 6x = 0$$

Q4. Write the equation in standard form. Then identify a, b, and c.

$$7 - 2x^2 = 0$$

Q5. Expand and write the equation in standard form. Then identify a, b, and c.

$$(x + 3)(x - 1) = 2x$$

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 2

The Quadratic Formula

A method for solving any quadratic equation.

Start with standard form

$$ax^2 + bx + c = 0, \quad a \neq 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Understand each part

$-b$	the opposite of b
$\pm$	calculate once with + and once with −
$b^2 - 4ac$	the discriminant
$2a$	divide the entire numerator by $2a$

Example:

$x^2 - 5x + 6 = 0$

$a = 1 \quad b = -5 \quad c = 6$

$$x = \frac{5 \pm \sqrt{25 - 24}}{2}$$

$$x = \frac{5 \pm 1}{2}$$

$x = 3$ or $x = 2$

Remember: Use parentheses when substituting negative numbers.

The quadratic formula

Q6. For the equation below, find the values of −b, $b^2 - 4ac$, and 2a. Then solve.

$$x^2 - 7x + 10 = 0$$

Q7. Solve using the quadratic formula. Check your answers.

$$x^2 + 3x - 10 = 0$$

Q8. Solve using the quadratic formula. Check your answers.

$$x^2 - 8x + 12 = 0$$

Q9. Solve using the quadratic formula. Check your answers.

$$x^2 + 2x - 15 = 0$$

Q10. A student solving $x^2 - 6x + 5 = 0$ wrote $x = (-6 \pm \sqrt{16})/2$. Find the error, then solve correctly.

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 3

Substituting into the Formula

Use parentheses to keep every sign correct.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Identify the coefficients

$$2x^2 - 3x - 2 = 0$$

$$a = 2 \qquad b = -3 \qquad c = -2$$

2. Substitute with parentheses

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2(2)}$$

3. Simplify carefully

$$-(-3) = 3$$
$$(-3)^2 = 9$$
$$-4(2)(-2) = +16$$

$$x = \frac{3 \pm \sqrt{9 + 16}}{4}$$
$$= \frac{3 \pm 5}{4}$$

4. Find both solutions

$$x = \frac{3 + 5}{4} = 2$$

$$x = \frac{3 - 5}{4} = -\frac{1}{2}$$

Be careful: $(-3)^2 = 9$, but $-3^2 = -9$.

Substituting into the formula

Q11. Solve using the quadratic formula. Check your answers.

$$3x^2 - 5x - 2 = 0$$

Q12. Solve using the quadratic formula. Check your answers.

$$2x^2 + 5x - 3 = 0$$

Q13. Solve using the quadratic formula. The leading coefficient is negative.

$$-x^2 + 4x + 5 = 0$$

Q14. Evaluate the discriminant $b^2 - 4ac$ when $a = -2$, $b = -3$, and $c = 5$.

Q15. Solve using the quadratic formula. Check your answers.

$$4x^2 - 4x - 3 = 0$$

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 4

Two Rational Solutions

A positive perfect-square discriminant gives two rational solutions.

Solve: $2x^2 - 7x + 3 = 0$

1. Identify a, *b*, and c

$a = 2 \qquad b = -7 \qquad c = 3$

2. Calculate the discriminant

$$\Delta = b^2 - 4ac$$

$$\Delta = (-7)^2 - 4(2)(3) = 49 - 24 = 25$$

25 is positive and a perfect square.

3. Apply the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{7 \pm \sqrt{25}}{4} = \frac{7 \pm 5}{4}$$

4. Calculate both answers

$$x = \frac{7+5}{4} = 3 \qquad x = \frac{7-5}{4} = \frac{1}{2}$$

Check: $2(3)^2 - 7(3) + 3 = 0$ | $2\left(\frac{1}{2}\right)^2 - 7\left(\frac{1}{2}\right) + 3 = 0$

Rational solutions can be integers or fractions.

Two rational solutions

Q16. Solve using the quadratic formula. Check your answers.

$$3x^2 - 10x + 3 = 0$$

Q17. Solve using the quadratic formula. Check your answers.

$$6x^2 + x - 2 = 0$$

Q18. Solve using the quadratic formula. Check your answers.

$$5x^2 - 9x - 2 = 0$$

Q19. Write the equation in standard form, then solve using the quadratic formula.

$$2x^2 = 9x - 4$$

Q20. Solve using the quadratic formula. Check your answers.

$$4x^2 + 4x - 15 = 0$$

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 5

Two Irrational Solutions

Simplify radicals and keep answers in exact form.

Solve: $x^2 - 4x + 1 = 0$

1. Identify the coefficients

$a = 1 \quad b = -4 \quad c = 1$

2. Calculate the discriminant

$$\Delta = (-4)^2 - 4(1)(1) = 16 - 4 = 12$$

12 is positive, but not a perfect square.

3. Apply the formula

$$x = \frac{4 \pm \sqrt{12}}{2}$$

Simplify the radical: $\sqrt{12} = \sqrt{4 \cdot 3} = 2\sqrt{3}$

$$x = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

4. Write both exact solutions

$x = 2 + \sqrt{3}$ $\quad$ $x = 2 - \sqrt{3}$

Approximate values: $x \approx 3.732$ or $x \approx 0.268$

Keep the radical for an exact answer. Round only when asked.

Two irrational solutions

Q21. Solve. Give exact answers in simplest radical form.

$$x^2 - 6x + 4 = 0$$

Q22. Solve. Give exact answers in simplest radical form.

$$x^2 + 2x - 4 = 0$$

Q23. Solve. Give exact answers in simplest radical form.

$$2x^2 - 4x - 1 = 0$$

Q24. Solve. Give exact answers, then approximate to the nearest hundredth.

$$x^2 - 3x - 5 = 0$$

Q25. Solve. Give exact answers, then approximate to the nearest hundredth.

$$3x^2 + 6x + 1 = 0$$

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 6

Understanding the Discriminant

Predict the number of real solutions before solving.

$$\Delta = b^2 - 4ac$$

For $ax^2 + bx + c = 0$, where $a \neq 0$

$\Delta > 0$: Two distinct real solutions

Example: $x^2 - 5x + 6 = 0$

Calculation: $\Delta = (-5)^2 - 4(1)(6) = 1$

Two real solutions: $x = 2$ and $x = 3$

$\Delta = 0$: One real solution

Example: $x^2 - 6x + 9 = 0$

Calculation: $\Delta = (-6)^2 - 4(1)(9) = 0$

One repeated solution: $x = 3$

$\Delta < 0$: No real solutions

Example: $x^2 + 2x + 5 = 0$

Calculation: $\Delta = 2^2 - 4(1)(5) = -16$

The square root of a negative number is not real.

Check the sign of Δ: positive → two, zero → one, negative → none.

Understanding the discriminant

Q26. Use the discriminant to find the number and type of real solutions. Do not solve.

$$x^2 - 4x - 12 = 0$$

Q27. Use the discriminant to find the number of real solutions. If there is exactly one, find it.

$$x^2 + 10x + 25 = 0$$

Q28. Use the discriminant to find the number of real solutions.

$$2x^2 + 3x + 4 = 0$$

Q29. Use the discriminant to find the number of real solutions. If there is exactly one, find it.

$$9x^2 - 12x + 4 = 0$$

Q30. For what values of c does the equation have two distinct real solutions?

$$x^2 - 6x + c = 0$$

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 7

Solutions & Graphs

The discriminant tells you how many x-intercepts a parabola has.

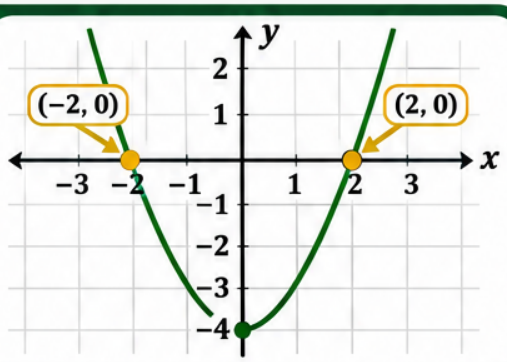
$$y = ax^2 + bx + c \qquad \Delta = b^2 - 4ac$$

$\Delta > 0$: Two x-intercepts

Equation: $y = x^2 - 4$

Calculation: $\Delta = 16$

Crosses the x-axis at $(-2, 0)$ *and* $(2, 0)$.

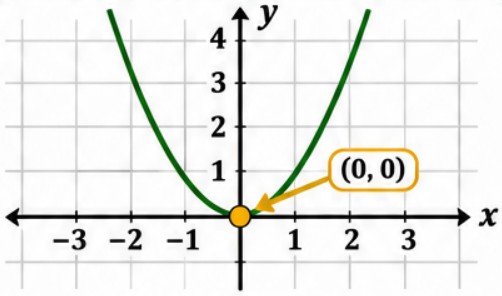


$\Delta = 0$: One x-intercept

Equation: $y = x^2$

Calculation: $\Delta = 0$

Touches the x-axis at $(0, 0)$.

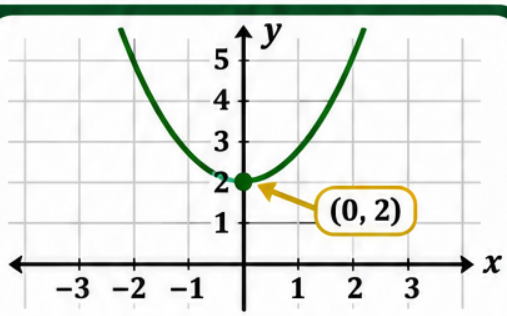


$\Delta < 0$: No x-intercepts

Equation: $y = x^2 + 2$

Calculation: $\Delta = -8$

Does not meet the x-axis.



Real solutions of $ax^2 + bx + c = 0$ *are the x-coordinates of the x-intercepts.*

Solutions and graphs: the discriminant and x-intercepts

Q31. How many x-intercepts does the parabola have? Find them.

$$y = x^2 - 2x - 8$$

Q32. How many x-intercepts does the parabola have? Find them.

$$y = x^2 + 4x + 4$$

Q33. How many x-intercepts does the parabola have? Explain using the graph's position.

$$y = x^2 - 2x + 5$$

Q34. How many x-intercepts does the parabola have? Find them.

$$y = -x^2 + 6x - 9$$

Q35. How many x-intercepts does the parabola have? Find them exactly and approximately.

$$y = 2x^2 - 3x - 1$$

SCHOOL OF MATH

ALGEBRA 1 • CLASS 20 • ITEM 8

Apply, Check & Interpret

Use the quadratic formula to solve a real-world problem.

A rectangular garden has an area of 40 m^2. Its length is 3 m more than its width. Find its dimensions.

1. Build the equation

Let w be the width. Then the length is $w + 3$.

$$w(w+3) = 40$$
$$w^2 + 3w - 40 = 0$$

2. Use the quadratic formula

$$a = 1 \qquad b = 3 \qquad c = -40$$

$$w = \frac{-3 \pm \sqrt{3^2 - 4(1)(-40)}}{2(1)}$$

$$w = \frac{-3 \pm \sqrt{169}}{2} = \frac{-3 \pm 13}{2}$$

3. Interpret both solutions

$$w = 5 \quad or \quad w = -8$$

A width must be positive, so $w = 5$ m.

Length $= 5 + 3 = 8$ m.

4. Check in the original problem

Area: $5 \times 8 = 40\ m^2$ ✓

Difference: $8 - 5 = 3$ m ✓

Width: 5 m Length: 8 m

A mathematical solution must also make sense in context.

Apply, check and interpret

Q36. A rectangle's length is 4 cm more than its width. Its area is 60 cm^2. Find its dimensions.

Q37. A ball is thrown upward from a 40-foot platform. Its height after t seconds is $h = -16t^2 + 24t + 40$. When does it hit the ground?

Q38. The product of two consecutive positive integers is 132. Find the integers.

Q39. One leg of a right triangle is 2 m longer than the other leg. The hypotenuse is 10 m. Find the lengths of the legs.

Q40. A rectangular rug's length is 2 ft more than its width, and its area is 20 ft^2. Find the exact width, then approximate both dimensions to the nearest hundredth.

Homework review

R1. Write the equation in standard form and identify a, b, and c.

$$6 - x = 3x^2$$

R2. Solve using the quadratic formula. Check your answers.

$$x^2 - 9x + 14 = 0$$

R3. Solve using the quadratic formula. Check your answers.

$$2x^2 - x - 6 = 0$$

R4. Solve using the quadratic formula. Check your answers.

$$10x^2 - 11x + 3 = 0$$

R5. Solve. Give exact answers and approximate to the nearest hundredth.

$$x^2 + 8x + 11 = 0$$

Homework review (continued)

R6. Find every value of k for which the equation has exactly one real solution.

$$x^2 + kx + 49 = 0$$

R7. Without graphing, how many x-intercepts does the parabola have?

$$y = x^2 - 5x + 7$$

R8. A rectangle's length is 6 in more than its width, and its area is 72 in². Find its dimensions.

Homework word problems

W1. The square of a number plus 3 times the number is 28. Find every such number.

W2. A rectangular poster has an area of 84 in². Its length is 5 in more than its width. Find its dimensions.

W3. The product of two consecutive positive even integers is 168. Find the integers.

W4. A stone is thrown upward from an 80-foot cliff. Its height after t seconds is $h = -16t^2 + 64t + 80$. When does it reach the ground?

W5. A triangle's base is 3 cm longer than its height, and its area is 27 cm². Find the height and the base.

Homework word problems (continued)

W6. An 8 in by 10 in photo has a frame of uniform width x around it. The photo and frame together cover 120 in². How wide is the frame?

W7. A theater sells 200 − 10p tickets when the price is p dollars. What ticket price brings in $750 in revenue?

W8. A ball is kicked from 6 ft above the ground. Its height after t seconds is $h = -16t^2 + 40t + 6$. When does it land? Give the exact time and round to the nearest hundredth.

W9. A ball's height after t seconds is $h = -16t^2 + 48t + 4$. Can it ever reach 50 feet? Use the discriminant.

W10. The diagonal of a rectangle is 13 m. The length is 7 m more than the width. Find the dimensions.

Homework word problems (continued)

W11. A 12 m by 16 m garden is surrounded by a path of uniform width. The garden and path together cover 320 m². How wide is the path?

W12. Challenge: Two numbers have a sum of 10 and a product of 20. Find them exactly. Then use the discriminant to find the largest possible product of two real numbers whose sum is 10.