



YOUNG FERMATS · ALGEBRA II · CLASS 13

Rational Expressions

Simplifying, multiplying,
and dividing

Student workbook

Eight concept images

40 practice questions · 20 homework questions

School of Math · Upper West Side

Understanding Rational Expressions

A **rational expression** is a fraction whose numerator and denominator are polynomials.

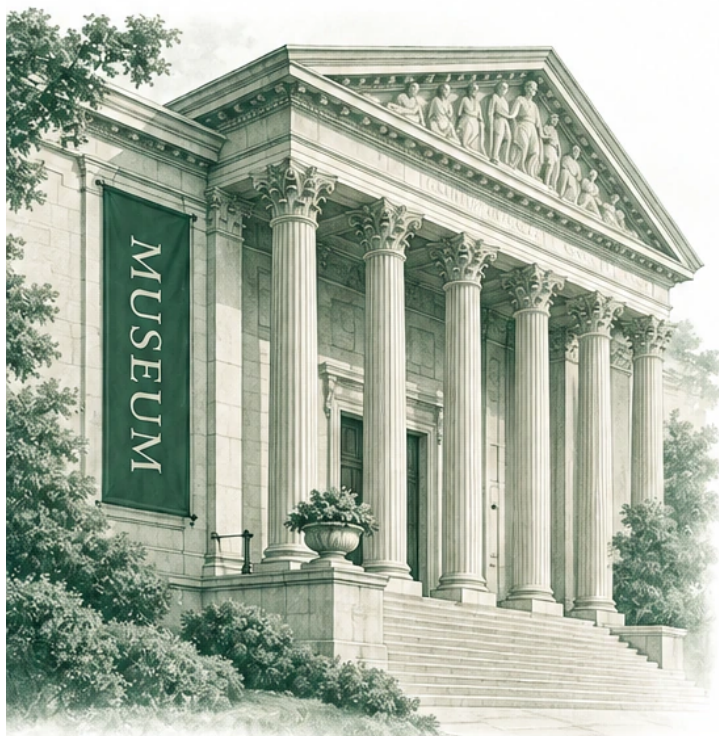
$$\frac{x + 2}{x - 3}$$

Numerator:
polynomial

Denominator:
polynomial

The denominator cannot equal zero.

Here, $x \neq 3$.



Real-world connection: A museum visit

Suppose a group pays a \$120 guide fee plus \$5 per student.

x = number of students.

Average cost per student =

$$\frac{120 + 5x}{x}$$

For 20 students:

$$\frac{120 + 5 \cdot 20}{20} = \$11 \text{ per student.}$$

In this context, x is a positive whole number.



MATH HINT

The numerator gives the total cost. Dividing by x gives the cost per student.

Image 1 practice: Understanding rational expressions

Q1. In $\frac{4x - 1}{x + 6}$, identify the numerator, denominator, and excluded value.

Q2. Evaluate $\frac{2x + 3}{x - 4}$ at $x = 6$.

Q3. Is $\frac{x - 7}{x + 1}$ zero or undefined at $x = 7$? Explain.

Q4. Which is a rational expression: $\frac{x^2 + 4}{x - 6}$ or $\frac{\sqrt{x} + 4}{x - 6}$?

Q5. An art workshop costs \$84 plus \$6 per student. Write the average cost for x students and evaluate it for 12 students.

Finding Excluded Values

Excluded values make the original denominator equal to zero. They are not allowed.

$$\frac{x + 1}{x^2 - 9}$$

1 Set the denominator equal to zero.

$$x^2 - 9 = 0$$

2 Factor.

$$(x - 3)(x + 3) = 0$$

3 Solve and exclude.

$$x = 3 \text{ or } x = -3$$

Restrictions: $x \neq 3$ and $x \neq -3$.

Domain: all real numbers except -3 and 3 .



Real-world connection: Cycling speed

A cyclist travels 12 miles in t hours.

$$\text{Average speed} = \frac{12}{t} \text{ mph.}$$

Here, $t \neq 0$.

Dividing by zero is undefined.

For this trip, elapsed time must be positive: $t > 0$.

If $t = 1.5$, the average speed is 8 mph.



MATH HINT

Check the **ORIGINAL** denominator before simplifying.
A canceled factor does not remove an excluded value.

Image 2 practice: Finding excluded values

Q6. Find the excluded value of $\frac{7}{x+4}$.

Q7. Find all excluded values of $\frac{x-1}{x(x-6)}$.

Q8. Find the real domain of $\frac{x+3}{x^2-49}$.

Q9. Find all excluded values of $\frac{x-5}{x^2-5x+6}$.

Q10. Find every excluded value of $\frac{x^2-4}{2x^2+2x-12}$. Does cancellation restore any value?

Factoring Before Simplifying

Factor the numerator and denominator completely to reveal common factors.

$$\frac{x^2 + 5x + 6}{x^2 - 4}$$

1

Factor the numerator.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$$2 + 3 = 5 \text{ and } 2 \times 3 = 6.$$

2

Factor the denominator.

$$x^2 - 4 = (x + 2)(x - 2)$$

Use the difference of squares.

3

Rewrite using factors.

$$\frac{(x + 2)(x + 3)}{(x + 2)(x - 2)}$$

Common factor: $x + 2$.

Original restrictions: $x \neq -2$ and $x \neq 2$.



Real-world connection: Garden design

A rectangular garden has area $x^2 + 5x + 6$ square meters and width $x + 2$ meters, where $x > 0$.

$$\text{Length} = \frac{x^2 + 5x + 6}{x + 2} = x + 3 \text{ meters.}$$

Factoring reveals the missing dimension.



MATH HINT

Look for a greatest common factor first, then check for trinomials and special patterns.

Image 3 practice: Factoring before simplifying

Q11. Simplify and state every excluded value: $\frac{6x + 18}{3}$.

Q12. Simplify and state every excluded value: $\frac{x^2 - 36}{x + 6}$.

Q13. Simplify and state every excluded value: $\frac{x^2 + 9x + 20}{x + 5}$.

Q14. Simplify and state every excluded value: $\frac{x^2 + 8x + 15}{x^2 - 9}$.

Q15. Simplify and state every excluded value: $\frac{2x^2 + 10x + 12}{4x^2 - 16}$.

Simplifying Rational Expressions

Cancel common factors, then keep every restriction from the original denominator.

$$\frac{(x+2)(x+3)}{(x+2)(x-2)}$$

1 Record the restrictions.

$$(x+2)(x-2) \neq 0$$

$$x \neq -2 \text{ and } x \neq 2.$$

2 Cancel the common factor.

$$\frac{\cancel{(x+2)}(x+3)}{\cancel{(x+2)}(x-2)}$$

$$\frac{x+3}{x-2} = 1 \text{ because } x \neq -2.$$

3 Write the simplified expression.

$$\frac{x+3}{x-2} \quad x \neq -2, 2$$

The expression changes form; the original restrictions remain.



Real-world connection: Printing posters

Suppose the total cost of printing n posters is $4n$ dollars, plus a setup fee of 20 dollars, where n is a positive whole number.

$$\text{Average cost} = \frac{4n + 20}{n} = 4 + \frac{20}{n}$$

Only the common factor in $4n/n$ cancels.
The setup fee is shared across the order.



MATH HINT

Cancel factors, not terms. Never cancel across addition or subtraction.

Image 4 practice: Simplifying and keeping restrictions

Q16. Simplify and state every excluded value: $\frac{10x^2}{15x}$.

Q17. Simplify and state every excluded value: $\frac{(x-5)(x+7)}{(x-5)(x+2)}$.

Q18. Simplify and state every excluded value: $\frac{x^2 - 64}{x^2 + 10x + 16}$.

Q19. Simplify and state every excluded value: $\frac{x^2 - 10x + 25}{x^2 - 25}$.

Q20. Simplify and state every excluded value: $\frac{3x^2 - 12x}{x^2 - 8x + 16}$.

Avoiding Common Cancellation Mistakes

Cancel common factors, never individual terms joined by addition or subtraction.

✗ INCORRECT

$$\frac{x + 3}{x} \neq 3$$

The x in $x + 3$ is a term, not a factor of the whole numerator.

✓ CORRECT

$$\frac{3x}{x} = 3, x \neq 0$$

The numerator is a product: $3 \times x$. The common factor x cancels.

1 Factor first when possible.

$$\frac{3x + 6}{3} = \frac{3(x + 2)}{3} = x + 2$$

The factor 3 multiplies the entire numerator.

2 Check with a number.

At $x = 2$: $\frac{x + 3}{x} = \frac{5}{2}$, not 3.



Real-world connection: Movie tickets

Suppose n movie tickets cost \$12 each, plus one \$6 booking fee per order.

$$\text{Average cost per ticket} = \frac{12n + 6}{n} = 12 + \frac{6}{n}$$

$$\text{For 3 tickets: } 12 + \frac{6}{3} = \$14 \text{ per ticket.}$$

Here, n is a positive whole number.



MATH HINT

Ask: Does this factor multiply the ENTIRE numerator and denominator?

Image 5 practice: Avoiding cancellation mistakes

Q21. A student says $\frac{x+8}{x} = 8$. Use $x = 4$ to check the claim.

Q22. Simplify $\frac{5x+15}{5}$. Explain why this cancellation is valid.

Q23. Rewrite $\frac{4x+12}{x}$ as a sum of two terms and state the restriction.

Q24. A student writes $\frac{x^2-25}{x-5} = x+5$ and says the original is valid at $x = 5$. What must be corrected?

Q25. Simplify $\frac{7-x}{x-7}$ and state the restriction.

Multiplying Rational Expressions

Factor completely, cancel common factors, then multiply what remains.

$$\frac{x^2 - 4}{x + 1} \cdot \frac{x + 1}{x + 2}$$

General rule:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

$$b \neq 0, d \neq 0$$

1

Record the restrictions.

Original denominators: $x + 1$ and $x + 2$
 $x \neq -1$ and $x \neq -2$.

2

Factor and cancel.

$$\frac{(x - 2)(x + 2)}{x + 1} \cdot \frac{x + 1}{x + 2}$$

The common factors $x + 1$ and $x + 2$ cancel.

3

Multiply the remaining factors.

$$x - 2$$

Restrictions remain: $x \neq -1, -2$.



Real-world connection: Travel time

A train travels d kilometers at v kilometers per hour.

Time = $\frac{d}{v}$ hours Distance = speed $\times$ time

$$v \cdot \frac{d}{v} = d$$

The factor v cancels. In this context, $d > 0$ and $v > 0$.



MATH HINT

Cancel common factors before multiplying to keep the algebra simpler.

Image 6 practice: Multiplying rational expressions

Q26. Simplify and state every excluded value: $\frac{3x}{5} \cdot \frac{10}{9x}$.

Q27. Simplify and state every excluded value: $\frac{x+4}{x-1} \cdot \frac{x-1}{x+7}$.

Q28. Simplify and state every excluded value: $\frac{x^2-36}{x+2} \cdot \frac{x+2}{x+6}$.

Q29. Simplify and state every excluded value: $\frac{x^2+7x+12}{x^2-16} \cdot \frac{x-4}{x+6}$.

Q30. Simplify and state every excluded value: $\frac{2x^2-8}{3x+9} \cdot \frac{x^2+6x+9}{x^2+x-2}$.

Dividing Rational Expressions

Multiply by the reciprocal of the divisor. The divisor must be defined and cannot equal zero.

$$\frac{x^2 - 9}{x + 2} \div \frac{x - 3}{x + 2}$$

General rule:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$$

$$b \neq 0, c \neq 0, d \neq 0$$

1 Find all restrictions.

Denominators: $x + 2 \neq 0 \rightarrow x \neq -2$.

Divisor cannot be zero: $x - 3 \neq 0 \rightarrow x \neq 3$.

2 Multiply by the reciprocal.

$$\frac{x^2 - 9}{x + 2} \cdot \frac{x + 2}{x - 3}$$

Flip only the second fraction.

3 Factor, cancel, and simplify.

$$\frac{(x - 3)(x + 3)}{x + 2} \cdot \frac{x + 2}{x - 3} = x + 3$$

$x + 3$, with $x \neq -2$ and $x \neq 3$.



Real-world connection: Filling bottles

You have 6 liters of water. Each bottle holds $\frac{3}{4}$ liter.

$$\text{Number of bottles} = 6 \div \frac{3}{4} = 6 \times \frac{4}{3} = 8$$

Dividing by a fraction tells us how many equal portions fit.



MATH HINT

Check both original denominators and the numerator of the divisor before simplifying.

Image 7 practice: Dividing rational expressions

Q31. Simplify and state every excluded value: $\frac{x}{6} \div \frac{x}{3}$.

Q32. Simplify and state every excluded value: $\frac{x+4}{x-2} \div \frac{x+4}{x+5}$.

Q33. Simplify and state every excluded value: $\frac{x^2-49}{x+3} \div \frac{x-7}{x+3}$.

Q34. Simplify and state every excluded value: $\frac{x^2+7x+10}{x^2-4} \div \frac{x+5}{x-2}$.

Q35. Simplify and state every excluded value: $\frac{x^2-9}{x^2+4x+3} \div \frac{x-3}{x+4}$.

Combining Multiplication and Division

Rewrite division as multiplication by a reciprocal, factor, and simplify.

$$\frac{x^2 - 1}{x} \cdot \frac{x}{x + 2} \div \frac{x - 1}{x + 2}$$

1 Record every restriction.

Original denominators: $x \neq 0$ and $x \neq -2$.
The divisor cannot be zero: $x \neq 1$.

2 Rewrite division and factor.

$$\frac{(x - 1)(x + 1)}{x} \cdot \frac{x}{x + 2} \cdot \frac{x + 2}{x - 1}$$

Take the reciprocal of the divisor only.

3 Cancel common factors.

Cancel x , $x + 2$, and $x - 1$.

$$x + 1$$

Restrictions remain: $x \neq 0, -2, 1$.

Check at $x = 3$: $\frac{8}{3} \cdot \frac{3}{5} \div \frac{2}{5} = 4$.



Real-world connection: Scaling a recipe

One batch uses $\frac{3}{4}$ cup of flour and makes 6 servings. How much flour is needed for 10 servings?

$$\frac{3}{4} \times 10 \div 6 = \frac{3}{4} \times \frac{10}{6} = \frac{5}{4} \text{ cups}$$

That is $1\frac{1}{4}$ cups of flour.



MATH HINT

Multiplication and division have equal priority. Work left to right, or rewrite division using reciprocals first.

Image 8 practice: Combining multiplication and division

Q36. Simplify and state every excluded value: $\frac{2x}{3} \cdot \frac{9}{x} \div \frac{4}{1}$.

Q37. Simplify and state every excluded value: $\frac{x+2}{x-3} \cdot \frac{x-3}{x+4} \div \frac{x+2}{x+5}$.

Q38. Simplify and state every excluded value: $\frac{x^2-36}{x} \cdot \frac{x}{x+1} \div \frac{x-6}{x+1}$.

Q39. Simplify and state every excluded value: $\frac{x^2+8x+15}{x^2-9} \cdot \frac{x-3}{x+7} \div \frac{x+5}{x+7}$.

Q40. Simplify and state every excluded value: $\frac{x^2-25}{x^2-1} \div \frac{x-5}{x+1} \cdot \frac{x-1}{x+2}$.

Homework review: R1–R5

R1. Evaluate $\frac{3x - 2}{x + 5}$ at $x = 2$ and state the excluded value.

R2. Find all excluded values of $\frac{x + 2}{x^2 - 2x - 15}$.

R3. Simplify and state every excluded value: $\frac{x^2 + 11x + 30}{x + 6}$.

R4. Simplify and state every excluded value: $\frac{x^2 - 81}{x^2 + 12x + 27}$.

R5. Explain why $\frac{2x + 10}{x}$ is not always 12. Rewrite it correctly.

Homework review: R6–R8

R6. Simplify and state every excluded value: $\frac{x^2 - 64}{x + 1} \cdot \frac{x + 1}{x + 8}$.

R7. Simplify and state every excluded value: $\frac{x^2 - 16}{x + 6} \div \frac{x - 4}{x + 6}$.

R8. Simplify and state every excluded value: $\frac{x^2 - 49}{x} \cdot \frac{x}{x + 2} \div \frac{x - 7}{x + 2}$.

Homework word problems: W1–W5

W1. A pottery club pays a \$90 studio fee plus \$7 per participant. Write the average cost for n participants. Find the average for 15 participants.

W2. A runner covers 18 miles in t hours. Write the average speed and calculate it for 3 hours. State a sensible domain.

W3. A rectangular mat has area $x^2 + 9x + 14$ square meters and width $x + 2$ meters, where $x > 0$. Find its length and then evaluate at $x = 3$.

W4. A square display has area x^2 square meters, and a 16-square-meter panel is removed. The remaining area is rearranged into a rectangle of width $x - 4$ meters, with $x > 4$. Find the new length.

W5. Printing n postcards costs $8n + 24$ dollars. A student says the average is always \$32 because the n cancels. Give the correct average and the value for 6 postcards.

Homework word problems: W6–W10

W6. A rectangular sign has length $\frac{x+4}{x+1}$ meters and width $\frac{3(x+1)}{x+2}$ meters, with $x > 0$. Find its area in simplest form.

W7. A machine works at $\frac{5x}{x+2}$ parts per minute for $\frac{3(x+2)}{x}$ minutes, where $x > 0$. How many parts does the model predict?

W8. A ribbon is $\frac{x^2 - 25}{x + 1}$ meters long. Each piece must be $\frac{x - 5}{x + 1}$ meter long, where x is a whole number greater than 5. How many pieces can be cut without waste?

W9. A tank holds $\frac{x^2 + 7x + 12}{x + 2}$ liters. Each container holds $\frac{x + 3}{x + 2}$ liters. For a positive whole number x , how many full containers can be filled? Evaluate at $x = 2$.

W10. A rectangular panel has area $\frac{x^2 - 9}{x + 4}$ square meters and width $\frac{x - 3}{x + 2}$ meters, where $x > 3$. Find its length and then its length at $x = 4$.

Homework word problems: W11–W12

W11. A recipe uses $\frac{x+3}{x+1}$ cups of flour for $x+3$ servings, with x a positive whole number. How much flour is needed for $2(x+1)$ servings at the same rate?

W12. A machine runs at $\frac{x^2-16}{x+1}$ meters of ribbon per minute for $\frac{x+1}{x+2}$ minutes. Each spool holds $\frac{x-4}{x+2}$ meter. If x is a whole number greater than 4, how many spools are filled? Check $x=6$.