



ALGEBRA 2

CLASS 16

RADICAL EQUATIONS & EXTRANEEOUS SOLUTIONS



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1. Introduction to Radical Equations

A **radical equation** contains a variable inside a radical.

EXAMPLES

1 $\sqrt{x + 3} = 5$

2 $\sqrt{2x - 1} = x$

The variable is under the radical sign.

Recognize radical equations and restrictions

Q1. Which equation is a radical equation? Explain the role of the variable.

$$\sqrt{x+8} = 6, \quad x + \sqrt{8} = 6$$

Q2. State the real-domain restriction. Also state the additional sign restriction imposed by the equation.

$$\sqrt{3x-6} = x-4$$

Q3. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x+9} = 5$$

Q4. Why is a candidate obtained by squaring not automatically a solution? Use 4 and -4 as an example.

Q5. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{2x+7} = 7$$

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2. Solving Basic Square Root Equations

To solve a square root equation, isolate the square root and then square both sides.

EXAMPLE

1 $\sqrt{x} = 5$

2 Square both sides:
 $(\sqrt{x})^2 = 5^2$

3 $x = 25$

4 Check: $\sqrt{25} = 5$ ✓

Takeaway:
Isolate the square root, then square both sides.

Basic square-root equations

Q6. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x} = 8$$

Q7. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x - 6} = 4$$

Q8. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{4x + 1} = 9$$

Q9. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{\frac{x}{3} + 2} = 5$$

Q10. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{7 - 2x} = 3$$

ALGEBRA 2 | CLASS 16**3. Isolating the Radical**

To solve a radical equation, isolate the radical expression on one side, then square both sides.

EXAMPLE

- 1** $\sqrt{x + 3} + 2 = 7$
- 2** Subtract 2: $\sqrt{x + 3} = 5$
- 3** Square both sides: $x + 3 = 25$
- 4** Subtract 3: $x = 22$
- 5** Check: $\sqrt{22 + 3} + 2 = 7$ ✓

Takeaway:

Isolate the radical before squaring.

Isolate the radical before squaring

Q11. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x+4} + 3 = 11$$

Q12. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$2\sqrt{2x-3} = 14$$

Q13. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$3\sqrt{x-1} - 5 = 10$$

Q14. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$-2\sqrt{x+1} + 8 = 0$$

Q15. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\frac{1}{2}\sqrt{5x+6} + 2 = 5$$

ALGEBRA 2 | CLASS 16**4. Equations with One Square Root**

To solve an equation with one square root, isolate the square root, then square both sides and check for extraneous solutions.

EXAMPLE

- 1** Start: $\sqrt{2x + 1} = x - 1$
- 2** Square: $2x + 1 = (x - 1)^2$
- 3** Simplify: $x^2 - 4x = 0$
- 4** Factor: $x(x - 4) = 0$
- 5** Candidates: $x = 0$ or $x = 4$
- 6** Check: $x = 0 \rightarrow 1 \neq -1$ ✗
 $x = 4 \rightarrow 3 = 3$ ✓

Solution: $x = 4$

A square root equal to a variable expression

Q16. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x + 12} = x$$

Q17. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{5x - 6} = x$$

Q18. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{3x + 10} = x + 2$$

Q19. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{2x + 9} = x + 1$$

Q20. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{4x + 13} = x + 3$$

ALGEBRA 2 | CLASS 16**5. Understanding Extraneous Solutions**

An extraneous solution is a candidate that does not satisfy the original equation.

EXAMPLE

- 1** Start: $\sqrt{x} = -3$
- 2** Square both sides: $x = 9$
- 3** Check in the original: $\sqrt{9} = 3 \neq -3$
- 4** Therefore: $x = 9$ is extraneous.

No real solution.

KEY TAKEAWAY

Squaring can make unequal values equal:

$$3^2 = (-3)^2.$$

Always check the original equation.

Understand extraneous solutions

Q21. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x+5} = -6$$

Q22. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x+6} = x$$

Q23. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{2x+3} = x$$

Q24. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x+20} = x-2$$

Q25. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{4x+5} = x-1$$

ALGEBRA 2 | CLASS 16**6. Checking Solutions**

Substitute each candidate into the original equation.

EXAMPLE

$$\sqrt{x + 2} = x$$

1 $x = 2$: $\sqrt{2 + 2} = 2 \rightarrow 2 = 2$ ✓

2 $x = -1$: $\sqrt{-1 + 2} = -1 \rightarrow 1 \neq -1$ ✗

Solution: $x = 2$

$x = -1$ is extraneous.

KEY TAKEAWAY

Always substitute each candidate into the original equation to check if it works.

Check every candidate in the original equation

Q26. A student lists $x = -5$ and $x = 6$. Determine which candidates solve the original equation and justify each decision.

$$\sqrt{x + 30} = x$$

Q27. A student lists $x = 2$ and $x = 5$. Check both candidates in the original equation.

$$\sqrt{7x - 10} = x$$

Q28. Solve and explain whether a negative value of x can be a valid solution here.

$$\sqrt{x + 1} = x + 1$$

Q29. Solve and explain why a negative candidate can survive the original-equation check.

$$\sqrt{x + 6} = -x$$

Q30. Find both candidates, then check each in the original equation. Do not check only the squared equation.

$$\sqrt{x + 24} = x - 4$$

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7. Equations with Two Radicals

Solve the equation and check the solution.

EXAMPLE

$$\sqrt{x+5} = \sqrt{x-1} + 2$$

- 1 Square: $x + 5 = x + 3 + 4\sqrt{x-1}$
- 2 Isolate: $2 = 4\sqrt{x-1}$
- 3 Square again: $\frac{1}{4} = x - 1$
- 4 Candidate: $x = \frac{5}{4}$
- 5 Check: $\sqrt{\frac{25}{4}} = \sqrt{\frac{1}{4}} + 2 \rightarrow \frac{5}{2} = \frac{5}{2} \checkmark$

Solution: $x = \frac{5}{4}$

KEY TAKEAWAY

When solving equations with radicals, isolate a radical, square both sides, and always check your solution.

Equations with two square roots

Q31. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{x+8} = \sqrt{x} + 2$$

Q32. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{2x+7} = \sqrt{x+1} + 2$$

Q33. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{x+7} = \sqrt{x-2} + 3$$

Q34. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{x+9} + \sqrt{x} = 5$$

Q35. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{3x+4} = \sqrt{x+10}$$

ALGEBRA 2 | CLASS 16**8. Solving Equations with Cube Roots**

Solve the equation and check the solution.

EXAMPLE

$$\sqrt[3]{x + 2} = 3$$

- 1** Cube both sides: $(\sqrt[3]{x + 2})^3 = 3^3$
- 2** Simplify: $x + 2 = 27$
- 3** Solve: $x = 25$
- 4** Check: $\sqrt[3]{25 + 2} = \sqrt[3]{27} = 3$ ✓

Solution: $x = 25$ **KEY TAKEAWAY**

Isolate the cube root, then cube both sides.

Real cube-root equations

Q36. Solve using real cube roots and check in the original equation.

$$\sqrt[3]{x-4} = 2$$

Q37. Solve using real cube roots and check in the original equation.

$$\sqrt[3]{2x+1} = -3$$

Q38. Solve using real cube roots and check in the original equation.

$$2\sqrt[3]{x+5} + 3 = 11$$

Q39. Solve using real cube roots and check in the original equation.

$$\sqrt[3]{3x-2} - 1 = -5$$

Q40. Solve using real cube roots and check in the original equation.

$$-3\sqrt[3]{\frac{x}{2}+4} + 4 = -2$$

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9. No Real Solution

A real square root is always nonnegative.

EXAMPLE

$$\sqrt{x + 4} = -2$$

1

Domain:

For real x , $\sqrt{x + 4}$ is defined when $x + 4 \geq 0$, so $x \geq -4$.

2

Key fact:

A real square root is always nonnegative, so $\sqrt{x + 4} \geq 0$.

3

Conclusion:

Therefore it cannot equal -2 .

4

Check the extraneous solution:

Squaring gives $x + 4 = 4$, so $x = 0$.

But $\sqrt{0 + 4} = \sqrt{4} = 2 \neq -2$. **✗**
Thus, $x = 0$ is an extraneous solution.

NO REAL SOLUTION.

KEY TAKEAWAY

Check each candidate in the original equation.

Recognize no-real-solution cases

Q41. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x+7} = -4$$

Q42. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$3\sqrt{2x-1} + 5 = 2$$

Q43. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{x-4} + \sqrt{9-x} = -2$$

Q44. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{x-2} = \sqrt{x+7}$$

Q45. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{3-x} = x-4$$



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10. Radical Equations in Real Life

The time t (in seconds) for an object to fall from rest is modeled by $t = \sqrt{\frac{h}{16}}$, where h is the height in feet. From what height does it fall in 3 seconds?

SOLUTION

- 1 Substitute $t = 3$: $3 = \sqrt{\frac{h}{16}}$
- 2 Square both sides: $9 = \frac{h}{16}$
- 3 Solve for h : $h = 144$ feet
- 4 Check: $\sqrt{\frac{144}{16}} = \sqrt{9} = 3$

HEIGHT: 144 FEET.

Model ignores air resistance.

KEY TAKEAWAY

Use radical equations to solve real-world problems.

Apply radical equations to models

Q46. In the model $t = \sqrt{h/16}$, t is fall time in seconds and h is height in feet. An object falls from rest for 4 seconds. Let x represent h ; find the height. Ignore air resistance.

$$\sqrt{\frac{x}{16}} = 4$$

Q47. A square has side length 12 centimeters. Its area x satisfies the equation below. Find the area with units.

$$\sqrt{x} = 12$$

Q48. A hypothetical speed model is $v = \sqrt{20d}$, with v in feet per second and d in feet. A speed of 30 feet per second is recorded. Let x represent d and find the distance.

$$\sqrt{20x} = 30$$

Q49. A right triangle has one leg 6 meters and hypotenuse 10 meters. The other leg x satisfies the equation below. Find its length and reject any physically impossible value.

$$\sqrt{x^2 + 36} = 10$$

Q50. A cube has edge length 5 centimeters. Let x be its volume in cubic centimeters. Use the equation below to find the volume.

$$\sqrt[3]{x} = 5$$

Homework review

R1. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{6x - 5} = 7$$

R2. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$4\sqrt{x + 2} - 1 = 19$$

R3. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{x + 42} = x$$

R4. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$\sqrt{9x - 20} = x$$

Homework review

R5. Solve the equation with two radicals. Show each squaring step and check all candidates.

$$\sqrt{x + 12} = \sqrt{x} + 2$$

R6. Solve using real cube roots and check in the original equation.

$$\sqrt[3]{4x - 3} = -5$$

R7. Solve over the real numbers. State the restrictions and check every candidate in the original equation.

$$2\sqrt{5x + 2} + 7 = 1$$

R8. Find the nonnegative value of x and explain what happens to the negative algebraic root.

$$\sqrt{x^2 + 25} = 13$$

Homework word problems

W1. A square mat has side length 9 feet. Its area is x square feet. Use the radical equation below to find x .

$$\sqrt{x} = 9$$

W2. Using $t = \sqrt{h/16}$, an object released from rest falls for 2.5 seconds. Let x be height in feet; find x . Ignore air resistance.

$$\sqrt{\frac{x}{16}} = \frac{5}{2}$$

W3. A hypothetical model connects speed v in meters per second to distance d in meters by $v = \sqrt{18d}$. Let x be d . Find the distance when $v = 36$.

$$\sqrt{18x} = 36$$

W4. A circle has diameter 10 centimeters. Its area x satisfies the equation below. Find the exact area in terms of π .

$$2\sqrt{\frac{x}{\pi}} = 10$$

Homework word problems

W5. A rectangular screen is 9 inches tall and has a 15-inch diagonal. Its width is x inches. Use the equation below to find the width.

$$\sqrt{x^2 + 81} = 15$$

W6. A cube-shaped container has edge length 4 meters. Its volume x satisfies the equation below. Find the volume.

$$\sqrt[3]{x} = 4$$

W7. A cube-shaped display has volume x cubic feet. A trim-length model is $L = 2$ times the cube root of x , minus 1, with L in feet. The trim length is 5 feet. Find the volume.

$$2\sqrt[3]{x} - 1 = 5$$

W8. Two square garden beds have areas x and $x + 20$ square meters. The larger bed's side is 2 meters longer. Find both areas using the equation below.

$$\sqrt{x + 20} = \sqrt{x} + 2$$

Homework word problems

W9. Two square panels have areas x and $x + 9$ square centimeters. Their side lengths add to 9 centimeters. Find their areas.

$$\sqrt{x + 9} + \sqrt{x} = 9$$

W10. Two design models give the same height: $H = \sqrt{4x + 4}$ and $H = x - 2$, where x is a nonnegative design parameter and H is in meters. Find x and the common height.

$$\sqrt{4x + 4} = x - 2$$

W11. A sensor model gives reading $R = 3\sqrt{t - 2} + 4$ for $t \geq 2$ seconds. Let x represent t . Can the reading equal 1? Explain why or why not.

$$3\sqrt{x - 2} + 4 = 1$$

W12. For a project of length x meters, two hypothetical cost models are $A = 20 + 4\sqrt{x}$ dollars and $B = 2x$ dollars, with $x \geq 0$. Find the exact length where the costs agree, and reject any extraneous candidate.

$$4\sqrt{x} + 20 = 2x$$