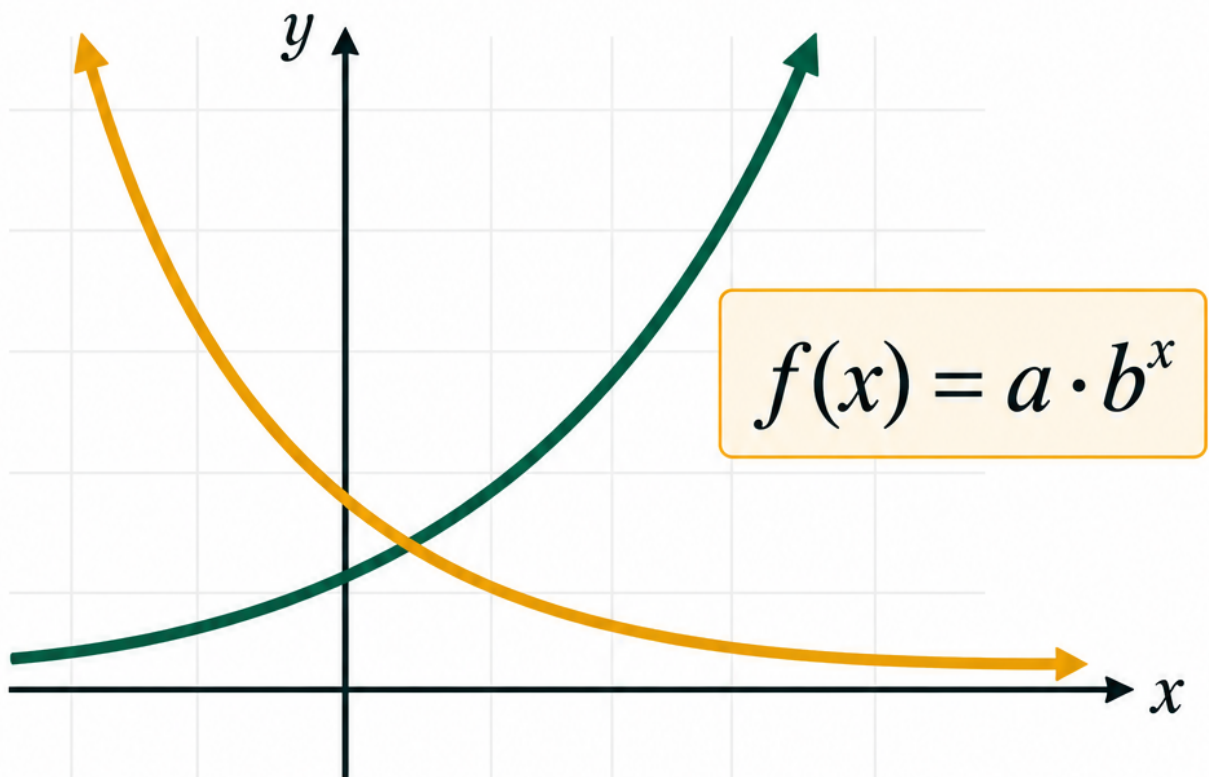




Exponential Functions

Growth, Decay, and Graphs

CLASS 17 • ALGEBRA 2





Introduction to Exponential Functions

CLASS 17 • ITEM 1 • ALGEBRA 2

An **exponential function** has a variable in the exponent.

$$f(x) = a \cdot b^x$$

a = initial value

b = base ($b > 0$, $b \neq 1$)

x = exponent

EXAMPLE

$$f(x) = 3 \cdot 2^x$$

x	0	1	2	3
$f(x)$	3	6	12	24



TAKEAWAY

The output is multiplied by 2 each time x increases by 1.

Recognizing exponential functions

Q1. Which functions are exponential? Explain each choice.

$$f(x) = 7 \cdot 4^x, \quad g(x) = 4x^3$$

Q2. Outputs at inputs 0, 1, 2, 3 are 6, 18, 54, 162. Show that an exponential model fits and write it.

Q3. Outputs at inputs 0, 1, 2, 3 are 8, 13, 18, 23. Are these values linear or exponential? Justify.

Q4. Evaluate $f(-1)$ exactly. Show the power before multiplying.

$$f(x) = 4 \cdot (3)^x$$

Q5. Explain why base 1 is excluded from a nonconstant exponential model.

$$f(x) = 12 \cdot 1^x$$



Initial Value and Growth Factor

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$$f(x) = a \cdot b^x$$

a = is the initial value: $f(0) = a$.

b = is the factor: multiply by b whenever x increases by 1.

EXAMPLE $f(x) = 5 \cdot 3^x$

Initial value:

$$a = 5$$

Growth factor:

$$b = 3$$

x	0	1	2	3
$f(x)$	5	15	45	135



TAKEAWAY

The output starts at 5 and triples at each step.

Initial value and repeated multiplier

Q6. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 7 \cdot (2)^x$$

Q7. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 18 \cdot \left(\frac{3}{4}\right)^x$$

Q8. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 125 \cdot \left(\frac{26}{25}\right)^x$$

Q9. An exponential model has $f(0) = 14$ and $f(1) = 35$. Find the base and write the model.

Q10. A positive-base exponential function passes through $(1, 12)$ and $(3, 108)$. Find a and b .

Exponential Growth

CLASS 17 • ITEM 3 • ALGEBRA 2

$$f(x) = a \cdot b^x$$

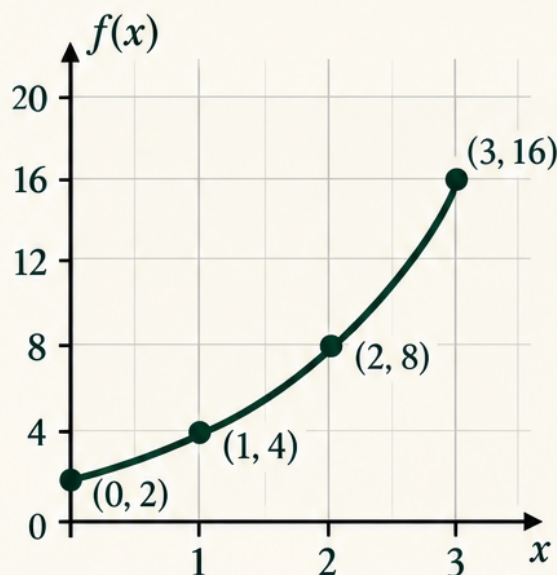
For $f(x) = a \cdot b^x$, when $a > 0$ and $b > 1$, the function shows exponential growth.

EXAMPLE

$$f(x) = 2 \cdot 2^x$$

x	$f(x)$
0	2
1	4
2	8
3	16

Each time x increases by 1, the output doubles.



TAKEAWAY

Exponential growth means multiplying by a factor greater than 1.

Exponential growth

Q11. Evaluate $f(4)$ exactly. Show the power before multiplying.

$$f(x) = 5 \cdot (2)^x$$

Q12. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 40 \cdot \left(\frac{5}{4}\right)^x$$

Q13. A model starts at 200 units and grows by 10% per year. Write an exponential model and find the amount after 3 years.

Q14. Compare the outputs after 3 periods. Which is larger, and by how much?

$$A(t) = 30 \cdot 2^t, \quad B(t) = 50(1.5)^t$$

Q15. A student models 20% growth from 80 units as $80(0.20)^t$. Explain the error and give the correct amount after 2 periods.

Exponential Decay

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$$f(x) = a \cdot b^x$$

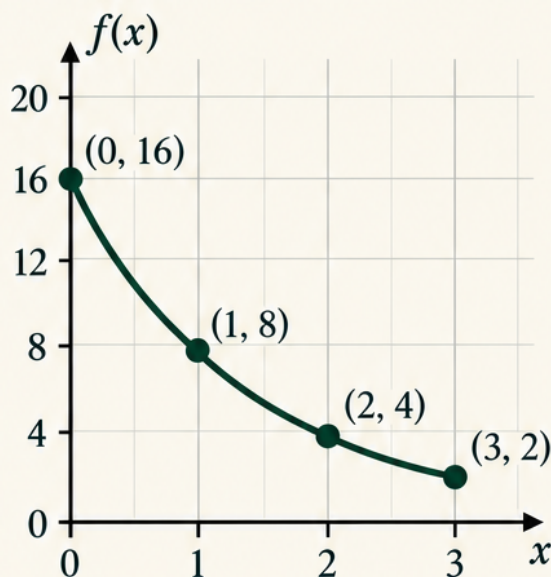
For $f(x) = a \cdot b^x$, when $a > 0$ and $0 < b < 1$, the function shows exponential decay.

EXAMPLE

$$f(x) = 16 \cdot \left(\frac{1}{2}\right)^x$$

x	$f(x)$
0	16
1	8
2	4
3	2

Each time x increases by 1, the output is cut in half.



TAKEAWAY

Exponential decay means multiplying by a factor between 0 and 1.

Exponential decay

Q16. Evaluate $f(3)$ exactly. Show the power before multiplying.

$$f(x) = 81 \cdot \left(\frac{1}{3}\right)^x$$

Q17. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 500 \cdot \left(\frac{22}{25}\right)^x$$

Q18. A sample starts at 320 grams and loses 25% of its mass each hour. Write an exponential model and find the amount after 2 hours.

Q19. A quantity halves every 3 hours, starting at 96 units. Write a model in hours and find the amount after 9 hours.

Q20. Will this model ever equal zero at a finite real input? Explain using the factors.

$$f(x) = 45(0.6)^x$$



Growth and Decay Rates

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To write an exponential function, convert a percentage rate r to a growth or decay factor b , with r expressed as a decimal.

GROWTH:
 $b = 1 + r$

For exponential growth,
add the rate r to 1.

EXAMPLE

8% growth:
 $r = 0.08 \rightarrow b = 1.08$

EXPONENTIAL FUNCTION

$$f(x) = a(1.08)^x$$

DECAY:
 $b = 1 - r$

For exponential decay,
subtract the rate r from 1.

EXAMPLE

8% decay:
 $r = 0.08 \rightarrow b = 0.92$

EXPONENTIAL FUNCTION

$$f(x) = a(0.92)^x$$



TAKEAWAY

Add the rate for growth;
subtract the rate for decay.

Converting rates and factors

Q21. A quantity increases by 12% each period. Find the multiplier and write a model with initial amount 240.

Q22. A quantity decreases by 18% each period. Find the multiplier and write a model with initial amount 240.

Q23. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 240 \cdot \left(\frac{207}{200}\right)^x$$

Q24. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 240 \cdot \left(\frac{37}{40}\right)^x$$

Q25. A quantity increases by 10% and then decreases by 10%. Find the overall multiplier and net percentage change.

Creating Tables of Values

CLASS 17 • ITEM 6 • ALGEBRA 2

Substitute each x -value into the function to find its output.

EXAMPLE

$$f(x) = 3 \cdot 2^x$$

CALCULATE EACH OUTPUT

$$f(0) = 3 \cdot 2^0 = 3$$

$$f(1) = 3 \cdot 2^1 = 6$$

$$f(2) = 3 \cdot 2^2 = 12$$

$$f(3) = 3 \cdot 2^3 = 24$$

TABLE OF VALUES

x	0	1	2	3
$f(x)$	3	6	12	24



TAKEAWAY

When x increases by 1, multiply the previous output by 2.

Tables of values and negative inputs

Q26. Make a table for $x = -2, -1, 0, 1, 2$. Explain the pattern between consecutive outputs.

$$f(x) = 2 \cdot (3)^x$$

Q27. Make a table for $x = -2, -1, 0, 1, 2$. Explain the pattern between consecutive outputs.

$$f(x) = 12 \cdot \left(\frac{1}{2}\right)^x$$

Q28. Make a table for $x = -2, -1, 0, 1, 2$. Explain the pattern between consecutive outputs.

$$f(x) = 6 \cdot (2)^x$$

Q29. Make a table for $x = -2, -1, 0, 1, 2$. Explain the pattern between consecutive outputs.

$$f(x) = 9 \cdot \left(\frac{1}{3}\right)^x$$

Q30. At inputs 0, 2, 4, the outputs are 5, 20, 80. Find the positive base per one-unit step and the missing outputs at $x = 1$ and $x = 3$.

Graphing Exponential Functions

CLASS 17 • ITEM 7 • ALGEBRA 2

- 1 Make a table of values.
- 2 Plot the points.
- 3 Draw a smooth curve.

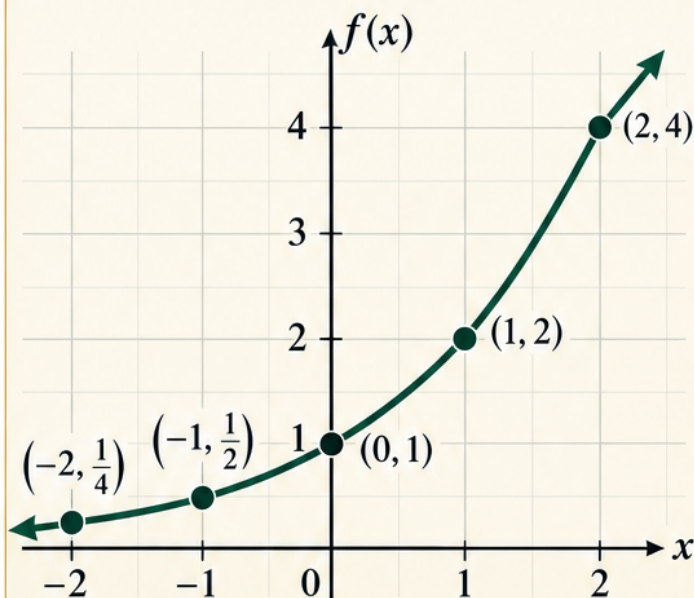
EXAMPLE

$$f(x) = 2^x$$

TABLE OF VALUES

x	$f(x) = 2^x$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

GRAPH OF $f(x) = 2^x$



TAKEAWAY

Plot points and sketch a smooth exponential curve.

Graphing from exact points

Q31. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = (3)^x$$

Q32. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 4 \cdot \left(\frac{1}{2}\right)^x$$

Q33. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 4 \cdot (2)^x$$

Q34. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 3 \cdot \left(\frac{1}{3}\right)^x$$

Q35. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = \left(\frac{1}{2}\right)^x$$

Key Features of Exponential Graphs

CLASS 17 • ITEM 8 • ALGEBRA 2

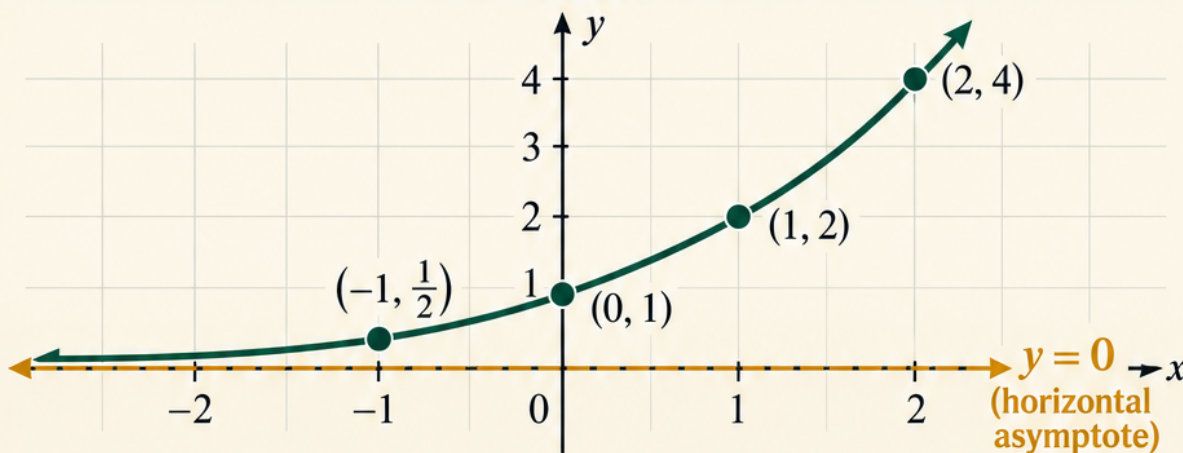
EXAMPLE

$$f(x) = 2^x$$

KEY FEATURES

- | | | |
|---|------------------------------|---------------------|
| 1 | y-intercept: | $(0, 1)$ |
| 2 | Domain: | $(-\infty, \infty)$ |
| 3 | Range: | $(0, \infty)$ |
| 4 | Horizontal asymptote: | $y = 0$ |

GRAPH OF $f(x) = 2^x$



TAKEAWAY

The graph approaches its horizontal asymptote.

Domain, range, and asymptotes

Q36. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 5 \cdot (2)^x$$

Q37. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 8 \cdot \left(\frac{1}{2}\right)^x$$

Q38. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = (3)^x + 2$$

Q39. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = -2 \cdot (2)^x$$

Q40. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 3 \cdot \left(\frac{1}{2}\right)^x - 1$$

Transformations of Exponential Functions

CLASS 17 • ITEM 9 • ALGEBRA 2

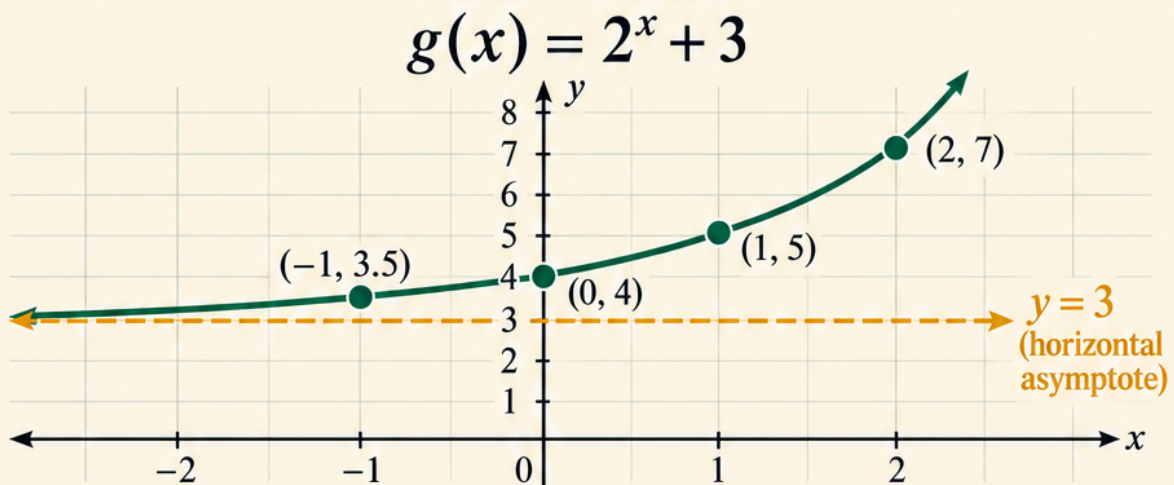
PARENT FUNCTION

$$f(x) = 2^x$$

KEY TRANSFORMATIONS

1	Vertical shift:	$2^x + 3$	→ up 3 units
2	Horizontal shift:	$2^{(x-2)}$	→ right 2 units
3	Reflection:	-2^x	→ reflect across the x -axis
4	Vertical stretch:	$3 \cdot 2^x$	→ multiply all y -values by 3

EXAMPLE: VERTICAL SHIFT



TAKEAWAY

Transformations change the position or shape of the parent graph.

Transformations of exponential graphs

Q41. Describe the transformations of the parent exponential, then sketch the graph and state its domain, range, y-intercept, and horizontal asymptote.

$$f(x) = (2)^x + 4$$

Q42. Describe the transformations of the parent exponential, then sketch the graph and state its domain, range, y-intercept, and horizontal asymptote.

$$f(x) = (2)^{x-1}$$

Q43. Describe the transformations of the parent exponential, then sketch the graph and state its domain, range, y-intercept, and horizontal asymptote.

$$f(x) = -(2)^x + 3$$

Q44. Describe the transformations of the parent exponential, then sketch the graph and state its domain, range, y-intercept, and horizontal asymptote.

$$f(x) = 3 \cdot (2)^{x+1} - 2$$

Q45. Describe the transformations of the parent exponential, then sketch the graph and state its domain, range, y-intercept, and horizontal asymptote.

$$f(x) = -2 \cdot \left(\frac{1}{2}\right)^{x-1} + 5$$



Real-World Applications

CLASS 17 • ITEM 10 • ALGEBRA 2

KEY IDEA

Exponential functions model repeated percentage change.

COMPOUND INTEREST EXAMPLE

A \$1,000 investment grows 5% per year, compounded annually.

$$A(t) = 1000(1.05)^t$$

t = years

$A(t)$ = value in dollars

CALCULATIONS

- | | | |
|----------|-----------------------|--|
| 1 | After 1 year: | $A(1) = 1000(1.05)^1$
$= 1000(1.05) = \$1,050.00$ |
| 2 | After 2 years: | $A(2) = 1000(1.05)^2$
$= 1000(1.1025) = \$1,102.50$ |

OTHER APPLICATIONS

Other applications: population growth and depreciation.



TAKEAWAY

The same percentage change is applied to the current amount each period.

Applications and compound interest

Q46. An account starts with \$1,200 and earns 4% nominal annual interest compounded annually. With no deposits or withdrawals, find its balance and interest earned after 3 years. Round only the final dollar amounts to cents.

Q47. A model population starts at 250 units and grows by 20% per day. Write an exponential model and find the amount after 2 days.

Q48. A car starts at \$18,000 and loses 15% of its value each year. Write an exponential model and find the amount after 2 years.

Q49. An account starts with \$2,000 and earns 6% nominal annual interest compounded quarterly. With no deposits or withdrawals, find its balance and interest earned after 2 years. Round only the final dollar amounts to cents.

Q50. An account starts with \$1,500 and earns 4.8% nominal annual interest compounded monthly. With no deposits or withdrawals, find its balance and interest earned after 1 year. Round only the final dollar amounts to cents.

Homework review

R1. Identify the initial value and multiplier. State growth or decay and the percentage change per one-unit increase in x .

$$f(x) = 64 \cdot \left(\frac{3}{4}\right)^x$$

R2. Evaluate $f(-3)$ exactly. Show the power before multiplying.

$$f(x) = 10 \cdot (2)^x$$

R3. A quantity increases by 2.5% each period. Find the multiplier and write a model with initial amount 240.

R4. Make a table for $x = -2, -1, 0, 1, 2$. Explain the pattern between consecutive outputs.

$$f(x) = 5 \cdot \left(\frac{1}{2}\right)^x$$

Homework review

R5. Sketch the graph and state its domain, range, y-intercept, horizontal asymptote, and increasing/decreasing behavior.

$$f(x) = 2 \cdot (3)^x - 4$$

R6. Describe the transformations of the parent exponential, then sketch the graph and state its domain, range, y-intercept, and horizontal asymptote.

$$f(x) = -3 \cdot (2)^{x-2} + 1$$

R7. An account starts with \$800 and earns 5% nominal annual interest compounded quarterly. With no deposits or withdrawals, find its balance and interest earned after 2 years. Round only the final dollar amounts to cents.

R8. An exponential model passes through (0, 20) and (2, 45). Find its equation and percentage growth per unit input.

Homework word problems

W1. A pattern starts with 40 tiles and doubles the number of tiles each round. Write an exponential model and find the amount after 3 rounds.

W2. A container begins with 600 milliliters. It loses 10% of the remaining liquid each hour. Write an exponential model and find the amount after 2 hours.

W3. A hypothetical account begins at \$350 and earns 8% annual interest, compounded annually, with no deposits or withdrawals. Write an exponential model and find the amount after 2 years.

W4. A laptop costs \$2,400 and loses 25% of its current value each year. Write an exponential model and find the amount after 3 years.

Homework word problems

W5. A culture has 150 cells at hour 0 and 600 cells at hour 2. Assuming constant exponential growth, find the hourly multiplier, a model, and the predicted count at hour 5.

W6. An 80-milligram substance has a half-life of 6 hours. Write its remaining-mass model and find the mass after 18 hours.

W7. An account starts with \$2,500 and earns 4% nominal annual interest compounded quarterly. With no deposits or withdrawals, find its balance and interest earned after 3 years. Round only the final dollar amounts to cents.

W8. Two hypothetical savings plans start at \$1,000. Plan A earns \$50 simple interest each year. Plan B grows by 5% each year, compounded annually. Compare balances after 4 years, with no deposits or withdrawals.

Homework word problems

W9. A cup's temperature in degrees Celsius follows the model below, where t is minutes after observation begins. Find its initial temperature, its predicted temperature after 3 minutes, and its long-run temperature.

$$T(t) = 22 + 58(0.8)^t$$

W10. A town's population model grows from 10,000 to 12,100 in 2 years. Find the annual growth rate and predict the population after a total of 3 years.

W11. Fund A starts at \$200 and doubles every year. Fund B starts at \$500 and grows by 25% per year. Assuming no deposits or withdrawals, determine the first whole year when Fund A exceeds Fund B.

W12. A \$3,000 machine loses 20% of its value each year. What is the first whole year when its value is below \$1,000? Show the two consecutive yearly values around the threshold.