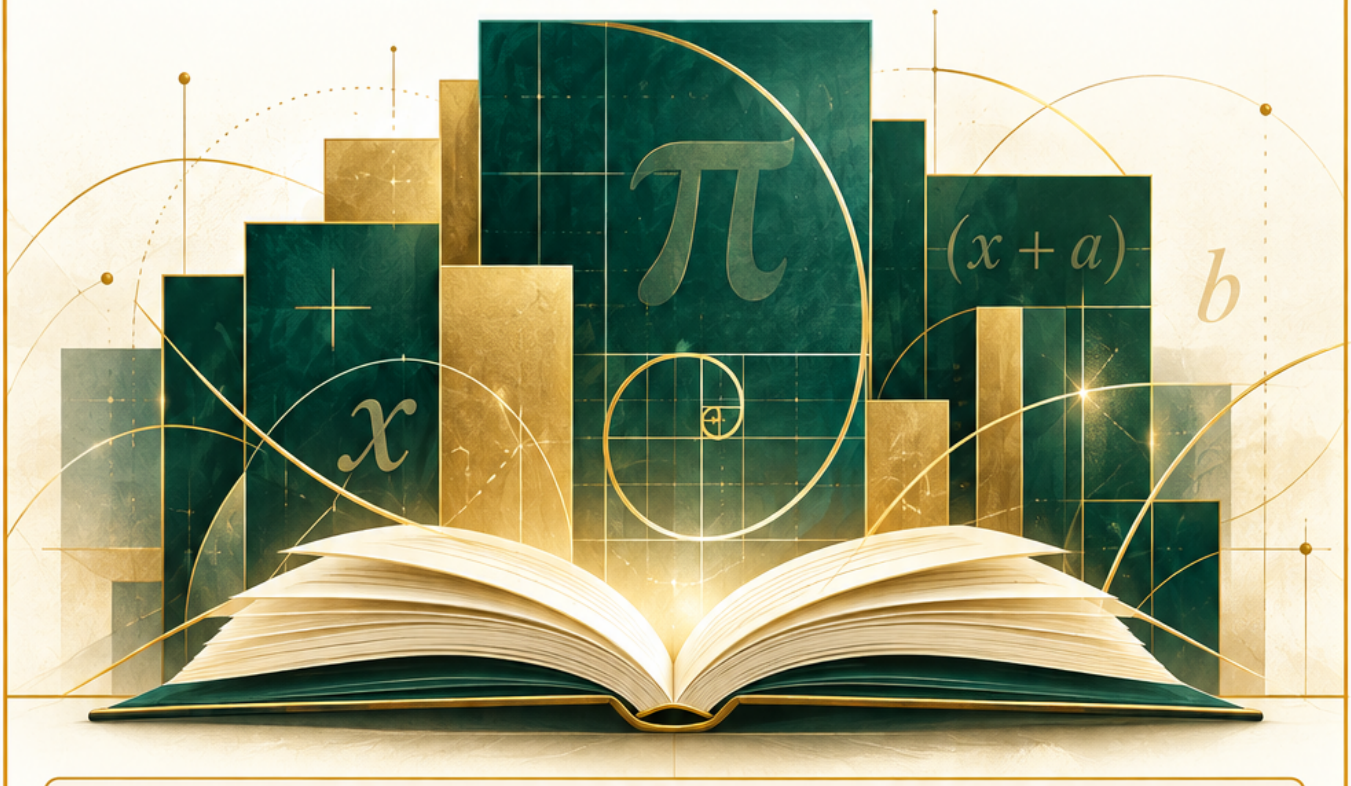
SCHOOL OF MATH

CLASS 13

# Factoring Trinomials

$$x^2 + bx + c$$

Recognize patterns. Find factors. Build confidence.




Sum & Product • Sign Patterns • Solving Equations

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# Recognizing a Trinomial

A trinomial is a polynomial with three terms.

General form:

$$x^2 + bx + c$$

The coefficient of $x^2$ is 1.

## EXAMPLE 1

$$x^2 + 7x + 12$$

| $x^2$ | $+7x$ | $+12$ |
|---|---|---|
| Quadratic term | Linear term | Constant term |
| coefficient: 1 | $b = 7$ | $c = 12$ |

Variable: $x$ | Highest exponent: 2 | Number of terms: 3

## WATCH THE SIGNS!

$$x^2 - 5x - 6$$

Terms: $x^2$, $-5x$, $-6$

$b = -5$ $c = -6$

## COUNT THE TERMS

$x^2 + 4x + 3$ → 3 terms → Trinomial

$x^2 + 3$ → 2 terms → Binomial

## TRY IT: IDENTIFY $b$ AND $c$

1. $x^2 + 9x + 20$

$b =$ _____ $c =$ _____

2. $x^2 - 2x - 15$

$b =$ _____ $c =$ _____

## TAKEAWAY

**Keep the sign with each term.**

# Factoring Trinomials

Image 1 practice: Recognizing a trinomial

**Q1.** In $x^2 + 12x + 35$, identify the quadratic term, linear term, constant term, b, and c.

**Q2.** For $x^2 - 13x + 36$, list all three terms and identify b and c.

**Q3.** Rewrite $18 + x^2 - 9x$ in descending powers of x. Then identify b and c.

**Q4.** Simplify $x^2 + 4x + 6x + 16$. Is the simplified result a trinomial? Identify b and c.

**Q5.** Compare $x^2 + 9x + 14$ and $2x^2 + 9x + 14$. Are both trinomials? Which has the form $x^2 + bx + c$, and how does it factor?

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# Connecting Multiplication and Factoring

**Multiplication expands an expression.**
**Factoring reverses the process.**

## MULTIPLY THE BINOMIALS

$$(x+3)(x+4)$$

| $x \cdot x = x^2$ | $x \cdot 4 = 4x$ | $3 \cdot x = 3x$ | $3 \cdot 4 = 12$ |
|---|---|---|---|
| First terms | Outside terms | Inside terms | Last terms |

$$x^2 + 4x + 3x + 12$$
$$= x^2 + 7x + 12$$

Combine like terms: $4x + 3x = 7x$.

## WHERE DO 7 AND 12 COME FROM?

| Add: | Multiply: |
|---|---|
| $3 + 4 = 7$ | $3 \times 4 = 12$ |

## REVERSE THE PROCESS

$$x^2 + 7x + 12 = (x+3)(x+4)$$

Factoring writes the trinomial as a product of binomials.

## THE GENERAL CONNECTION

$$(x+m)(x+n) = x^2 + (m+n)x + mn$$

$b = m + n$ | $c = mn$

## TRY IT

$$(x+2)(x+5) = x^2 + \_\_\_\_\,x + \_\_\_\_$$

Then write the trinomial in factored form.

## TAKEAWAY

**Expand to multiply. Reverse to factor.**

# Factoring Trinomials

Image 2 practice: Connecting multiplication and factoring

**Q6.** Expand $(x + 1)(x + 8)$, then write your trinomial in factored form again.

**Q7.** Expand $(x + 2)(x + 9)$. Explain where the middle coefficient and constant come from.

**Q8.** Expand $(x - 2)(x + 7)$, keeping track of the negative sign.

**Q9.** Expand $(x - 4)(x - 8)$. Explain why the constant is positive but the linear term is negative.

**Q10.** A rectangle has sides $x + 3$ and $x + 9$ units. Write its area in expanded and factored forms, then check both forms at $x = 2$.

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# The Sum-and-Product Rule

To factor $x^2 + bx + c$, find two numbers $m$ and $n$.

**SUM:** $m + n = b$

**PRODUCT:** $mn = c$

$$x^2 + bx + c = (x + m)(x + n)$$

## EXAMPLE: FACTOR $x^2 + 8x + 15$

1. Identify the targets: $b = 8$ and $c = 15$.
2. List positive factor pairs of 15.

| Pair | Product | Sum |
|---|---|---|
| 1 and 15 | 15 | 16 |
| ✓ 3 and 5 | 15 | 8 |

3. Choose 3 and 5: their sum is 8 and their product is 15.

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

## CHECK BY EXPANDING

$$\begin{aligned}(x + 3)(x + 5) &= x^2 + 5x + 3x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

## TRY IT

1. $x^2 + 7x + 10$
   Numbers: ___ and ___
   Factors: $(x + \_\_\_)(x + \_\_\_)$

2. $x^2 + 11x + 24$
   Numbers: ___ and ___
   Factors: $(x + \_\_\_)(x + \_\_\_)$

## TAKEAWAY

**Both conditions must work: add to $b$ and multiply to $c$.**

# Factoring Trinomials

Image 3 practice: The sum-and-product rule

**Q11.** Find two positive integers with sum 12 and product 32. Use them to factor $x^2 + 12x + 32$.

**Q12.** List all positive factor pairs of 40. Which pair factors $x^2 + 13x + 40$?

**Q13.** Find integers with sum −10 and product 21. Then factor $x^2 - 10x + 21$.

**Q14.** Factor $x^2 + 5x - 24$. Explain why the positive number must have the larger absolute value.

**Q15.** The trinomial $x^2 + bx + 45$ has a factor $x + 5$. Find its other factor and the value of $b$.

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# Factoring with Positive Signs

When $b > 0$ and $c > 0$, both numbers in the factors are positive.

$$x^2 + bx + c = (x + m)(x + n)$$
$$m + n = b \qquad mn = c$$

A positive product means matching signs. A positive sum means both are positive.

## EXAMPLE 1: FACTOR $x^2 + 9x + 20$

Find two positive numbers that multiply to 20 and add to 9.

| Pair | Product | Sum |
|---|---|---|
| 1 and 20 | 20 | 21 |
| 2 and 10 | 20 | 12 |
| ✓ 4 and 5 | 20 | 9 |

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

Check: $x^2 + 5x + 4x + 20 = x^2 + 9x + 20$

## EXAMPLE 2: FACTOR $x^2 + 10x + 21$

$3 + 7 = 10$ $\qquad$ $3 \times 7 = 21$

$$x^2 + 10x + 21 = (x + 3)(x + 7)$$

## TRY IT

1. $x^2 + 8x + 12$
   Numbers: ___ and ___
   $(x + \_\_\_)(x + \_\_\_)$

2. $x^2 + 11x + 30$
   Numbers: ___ and ___
   $(x + \_\_\_)(x + \_\_\_)$

## TAKEAWAY

**Positive sum + positive product → two positive numbers.**

# Factoring Trinomials

Image 4 practice: Factoring with positive signs

**Q16.** Factor completely: $x^2 + 12x + 20$. Show the sum and product you use.

**Q17.** Factor completely: $x^2 + 13x + 30$. Show the sum and product you use.

**Q18.** Factor completely: $x^2 + 15x + 44$. Show the sum and product you use.

**Q19.** Factor completely: $x^2 + 17x + 60$. Show the sum and product you use.

**Q20.** Factor completely: $x^2 + 19x + 88$. Show the sum and product you use.

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# Negative $b$, Positive $c$

**When $b < 0$ and $c > 0$, choose two negative numbers.**

| **Negative sum:** $m + n = b$ | **Positive product:** $mn = c$ |
|---|---|

A positive product requires matching signs. Since the sum is negative, both numbers are negative.

## EXAMPLE 1: FACTOR $x^2 - 7x + 12$

Find two numbers that multiply to +12 and add to −7.

| Pair | Product | Sum |
|---|---|---|
| −1 and −12 | +12 | −13 |
| −2 and −6 | +12 | −8 |
| ✓ −3 and −4 | +12 | −7 |

$$x^2 - 7x + 12 = (x - 3)(x - 4)$$

**Check:** $x^2 - 4x - 3x + 12 = x^2 - 7x + 12$

## EXAMPLE 2: FACTOR $x^2 - 9x + 20$

$(-4) + (-5) = -9$ $\qquad$ $(-4)(-5) = +20$

$$x^2 - 9x + 20 = (x - 4)(x - 5)$$

Use the signs: 3 and 4 add to +7; −3 and −4 add to −7.

## TRY IT

1. $x^2 - 8x + 15$
   Numbers: ___ and ___
   Factors: $(x - \_\_\_)(x - \_\_\_)$

2. $x^2 - 11x + 24$
   Numbers: ___ and ___
   Factors: $(x - \_\_\_)(x - \_\_\_)$

## TAKEAWAY

**Negative sum and positive product → two negative numbers.**

# Factoring Trinomials

Image 5 practice: Negative b, positive c

**Q21.** Factor completely: $x^2 - 9x + 14$. Show the sum and product you use.

**Q22.** Factor completely: $x^2 - 13x + 30$. Show the sum and product you use.

**Q23.** Factor completely: $x^2 - 13x + 36$. Show the sum and product you use.

**Q24.** Factor completely: $x^2 - 16x + 55$. Show the sum and product you use.

**Q25.** Factor completely: $x^2 - 20x + 96$. Show the sum and product you use.

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# Factoring When $c$ Is Negative

**A negative product means the two numbers have opposite signs.**

| $m + n = b$ | $mn = c < 0$ |
|---|---|

The number with the greater absolute value has the same sign as b.

## EXAMPLE 1: POSITIVE $b$

$$x^2 + x - 12$$

Find two numbers with product $-12$ and sum $+1$.

| $4 + (-3) = 1$ | $4(-3) = -12$ |
|---|---|

$$x^2 + x - 12 = (x + 4)(x - 3)$$

4 has the greater absolute value, so the sum is positive.

**Check:** $x^2 - 3x + 4x - 12 = x^2 + x - 12$

## EXAMPLE 2: NEGATIVE $b$

$$x^2 - x - 12$$

Find two numbers with product $-12$ and sum $-1$.

| $(-4) + 3 = -1$ | $(-4)(3) = -12$ |
|---|---|

$$x^2 - x - 12 = (x - 4)(x + 3)$$

$-4$ has the greater absolute value, so the sum is negative.

**Check:** $x^2 + 3x - 4x - 12 = x^2 - x - 12$

## TRY IT

**1.** $x^2 + 2x - 15$

Numbers: ___ and ___

Factored form: _____

**2.** $x^2 - 2x - 15$

Numbers: ___ and ___

Factored form: _____

## TAKEAWAY

**Negative $c$ → opposite signs. Use $b$ to choose which number is negative.**

# Factoring Trinomials

Image 6 practice: Factoring when c is negative

**Q26.** Factor completely: $x^2 + 4x - 5$. Show the sum and product you use.

**Q27.** Factor completely: $x^2 + 5x - 14$. Show the sum and product you use.

**Q28.** Factor completely: $x^2 - 5x - 24$. Show the sum and product you use.

**Q29.** Factor completely: $x^2 + 5x - 66$. Show the sum and product you use.

**Q30.** Factor completely: $x^2 - 6x - 91$. Show the sum and product you use.

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# Recognizing Special Cases

## 1. PERFECT-SQUARE TRINOMIALS

When the two numbers are equal, the factors repeat.

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

### EXAMPLE A

$$x^2 + 6x + 9$$

$3 + 3 = 6 \qquad 3 \times 3 = 9$

$$x^2 + 6x + 9 = (x + 3)^2$$

### EXAMPLE B

$$x^2 - 8x + 16$$

$(-4) + (-4) = -8 \qquad (-4)(-4) = 16$

$$x^2 - 8x + 16 = (x - 4)^2$$

**Check both:** the constant is a square AND the middle coefficient is twice the repeated number.

## 2. WHEN NO INTEGER PAIR WORKS

$$x^2 + x + 1$$

We need two integers with product 1 and sum 1.

| Pair | Product | Sum |
|---|---|---|
| 1 and 1 | 1 | 2 |
| −1 and −1 | 1 | −2 |

Neither pair adds to 1, so $x^2 + x + 1$ cannot be factored into linear factors with integer coefficients.

## TRY IT: FACTOR USING INTEGERS, IF POSSIBLE

1. $x^2 + 10x + 25$

   Factored form: _____

2. $x^2 + 3x + 5$

   Factored form: _____

## TAKEAWAY

**Repeated pair → perfect square. No integer pair → not factorable over the integers.**

# Factoring Trinomials

Image 7 practice: Special cases: squares and integer limits

**Q31.** Factor $x^2 + 12x + 36$ and write the result using an exponent.

**Q32.** Factor $x^2 - 14x + 49$. Explain the sign inside the squared factor.

**Q33.** Can $x^2 + 4x + 7$ be factored into linear factors with integer coefficients? Justify your answer.

**Q34.** A student calls $x^2 + 7x + 16$ a perfect square because 16 is a square. Is this correct? Can it factor over the integers?

**Q35.** Find c so that $x^2 - 18x + c$ is a perfect-square trinomial. Then factor it and verify the middle term.

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# Checking Factors and Solving Equations

## 1. CHECK BY EXPANDING

Proposed factorization: $x^2 + 5x + 6 = (x + 2)(x + 3)$

$$(x + 2)(x + 3) = x^2 + 3x + 2x + 6$$
$$= x^2 + 5x + 6$$

The expansion matches the original trinomial.

## ZERO PRODUCT PROPERTY

***If $AB = 0$, then $A = 0$ or $B = 0$.***

Use this rule only when the product equals zero.

## 2. SOLVE THE EQUATION

**Start:** $x^2 + 5x + 6 = 0$

**Factor:** $(x + 2)(x + 3) = 0$

$x + 2 = 0$
$x = -2$

**or**

$x + 3 = 0$
$x = -3$

**Solutions:** $x = -2$ or $x = -3$.

## 3. CHECK THE SOLUTIONS

$x = -2$:
$(-2)^2 + 5(-2) + 6 = 4 - 10 + 6$
$= 0$

$x = -3$:
$(-3)^2 + 5(-3) + 6 = 9 - 15 + 6$
$= 0$

## SET THE EQUATION EQUAL TO ZERO FIRST.

**Example:** $x^2 + 5x = -6 \rightarrow x^2 + 5x + 6 = 0$.

## TRY IT: FACTOR AND SOLVE

1. $x^2 + 7x + 12 = 0$
   Solutions: _____

2. $x^2 - x - 6 = 0$
   Solutions: _____

## TAKEAWAY

**Factor → set each factor equal to zero → solve → check.**

# Factoring Trinomials

Image 8 practice: Checking factors and solving equations

**Q36.** Check the claim $x^2 + 14x + 24 = (x + 2)(x + 12)$ by expanding the right side.

**Q37.** Is $(x - 7)(x - 4)$ a correct factorization of $x^2 - 3x - 28$? Expand, then correct it if needed.

**Q38.** Factor and solve $x^2 - 15x + 54 = 0$. Check both solutions.

**Q39.** Solve $x^2 + 4x = 45$ by factoring. Explain the first step.

**Q40.** Solve $x^2 - 6x = 40$ by factoring, and explain why neither $x = 0$ nor $x = 6$ can be assumed.

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# Check for a Greatest Common Factor First

Before factoring a trinomial, check whether every term shares a common factor.

**Factor out the GCF, then factor the trinomial inside the parentheses.**

## EXAMPLE 1: A NUMERICAL GCF

**Factor:** $2x^2 + 10x + 12$

**Step 1:** The GCF of 2, 10, and 12 is 2.

**Step 2:** Divide every term by 2:

$$2x^2 + 10x + 12 = 2(x^2 + 5x + 6)$$

**Step 3:** Find two numbers with sum 5 and product 6: 2 and 3.

$$2x^2 + 10x + 12 = 2(x + 2)(x + 3)$$

## EXAMPLE 2: A VARIABLE GCF

**Factor:** $x^3 + 7x^2 + 12x$

**Step 1:** Every term contains $x$. The GCF is $x$.

**Step 2:** $x^3 + 7x^2 + 12x = x(x^2 + 7x + 12)$

**Step 3:** $3 + 4 = 7 \qquad 3 \times 4 = 12$

$$x^3 + 7x^2 + 12x = x(x + 3)(x + 4)$$

After removing the GCF, the trinomial has leading coefficient 1.

## KEEP THE GCF!

The common factor remains outside the parentheses in your final answer.

**Check:** $2(x + 2)(x + 3) = 2(x^2 + 5x + 6) = 2x^2 + 10x + 12$

## TRY IT: FACTOR COMPLETELY

1. $3x^2 + 15x + 18$
   GCF: _____
   Factored form: _____

2. $x^3 - 5x^2 + 6x$
   GCF: _____
   Factored form: _____

## TAKEAWAY

**GCF first → factor the trinomial → keep every factor.**

# Factoring Trinomials

Image 9 practice: Taking out the greatest common factor

**Q41.** Factor completely: $2x^2 + 26x + 80$. Show the sum and product you use.

**Q42.** Factor completely: $3x^2 - 42x + 135$. Show the sum and product you use.

**Q43.** Factor completely: $x^3 + 3x^2 - 40x$. Show the sum and product you use.

**Q44.** Factor completely: $4x^3 - 44x^2 + 112x$. Show the sum and product you use.

**Q45.** Factor completely: $-2x^2 + 26x - 84$. Show the sum and product you use.

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# Recognizing and Correcting Common Mistakes

A correct factorization must reproduce every term of the original expression.

## 1. CHECKING ONLY THE SUM

**✗ Incorrect:** $x^2 + 8x + 15 \neq (x + 2)(x + 6)$

$2 + 6 = 8$, but $2 \times 6 = 12$, not 15.

**✓ Correct:** $x^2 + 8x + 15 = (x + 3)(x + 5)$

$3 + 5 = 8$ and $3 \times 5 = 15$.

## 2. CHOOSING THE WRONG SIGNS

**✗ Incorrect:** $x^2 - 7x + 12 \neq (x + 3)(x + 4)$

Positive 3 and 4 produce $+7x$, not $-7x$.

**✓ Correct:** $x^2 - 7x + 12 = (x - 3)(x - 4)$

$(-3) + (-4) = -7$ and $(-3)(-4) = 12$.

## 3. DROPPING THE GCF

**✗ Incorrect:** $2x^2 + 10x + 12 \neq (x + 2)(x + 3)$

The common factor 2 must remain in the answer.

**✓ Correct:** $2x^2 + 10x + 12 = 2(x + 2)(x + 3)$

## FINAL CHECKLIST

- ☑ Check the GCF.
- ☑ Check the sum and product.
- ☑ Check the signs.
- ☑ Expand to verify.

## SPOT AND FIX THE ERROR

Incorrect student work:

$x^2 + 7x + 10 = (x + 1)(x + 10)$.

What is wrong? _____

Correct factorization: _____

## TAKEAWAY

**Both conditions must work: add to $b$ and multiply to $c$.**

# Factoring Trinomials

Image 10 practice: Recognizing and correcting common mistakes

**Q46.** A student writes $x^2 + 16x + 48 = (x + 6)(x + 10)$. Which test fails? Correct the factorization.

**Q47.** Correct $x^2 - 17x + 72 = (x + 8)(x + 9)$. Explain the sign error.

**Q48.** A student factors $x^2 + 6x - 40$ as $(x - 10)(x + 4)$. Explain what went wrong and correct it.

**Q49.** A student writes $5x^2 + 75x + 250 = (x + 5)(x + 10)$. Correct the work and verify the leading coefficient.

**Q50.** For $x^2 + 9x + 14 = 14$, a student reports $x = -2$ or $-7$ after factoring the left side. Explain the error and solve correctly.

# Factoring Trinomials

Homework review: R1–R5

**R1.** Image 1: Simplify $22 + x^2 - 5x - 8x$, write it in standard form, and identify b and c.

**R2.** Image 2: Expand $(x - 3)(x + 11)$, then explain how the sum and product of the constants appear in your result.

**R3.** Image 3: Find the integer pair for $x^2 + 15x + 56$, then factor it. Show both target checks.

**R4.** Image 4: Factor completely: $x^2 + 17x + 66$. Show the sum and product you use.

**R5.** Image 5: Factor completely: $x^2 - 17x + 70$. Show the sum and product you use.

# Factoring Trinomials

Homework review: R6–R10

**R6.** Image 6: Factor completely: $x^2 + 4x - 45$. Show the sum and product you use.

**R7.** Image 7: Factor $x^2 - 22x + 121$. Explain why it is a perfect square.

**R8.** Image 8: Solve $x^2 + 2x = 63$ by factoring. Check both values.

**R9.** Image 9: Factor completely: $6x^3 + 30x^2 - 396x$. Show the sum and product you use.

**R10.** Image 10: A student claims $x^2 - 19x + 84 = (x - 6)(x - 14)$. Identify the failed check and correct the factors.

# Factoring Trinomials

Homework word problems: W1–W5

**W1.** A rectangular sign has area $x^2 + 14x + 45$ square centimeters. Its sides have the form $x + m$ and $x + n$, with positive integer constants. Factor the area to find the two side expressions.

**W2.** A garden has area $x^2 + 16x + 63$ square meters and length $x + 9$ meters. Find its width by factoring. Then find both dimensions when $x = 2$.

**W3.** A puzzle asks for two positive whole numbers whose sum is 17 and product is 70. Use factoring to find the numbers, and explain why a pair with only the correct sum is not enough.

**W4.** A rectangular print has area $x^2 - 15x + 56$ square inches, with $x > 8$. Its sides are $x$ minus positive integers. Factor to find the dimensions, then calculate its perimeter when $x = 12$.

**W5.** A rectangular stage has area $x^2 + 3x - 54$ square meters and length $x + 9$ meters. Find the width, state the restriction on $x$ for positive dimensions, and calculate the area at $x = 10$.

# Factoring Trinomials

Homework word problems: W6–W10

**W6.** A square mural has area $x^2 + 18x + 81$ square feet, where x is nonnegative. Factor the expression to find its side length. What is the perimeter when x = 4?

**W7.** Three identical rectangular panels have total area $3x^2 + 57x + 270$ square meters. Each panel has sides x plus positive integers. Factor the total area to find each panel's dimensions. Find the total area at x = 1.

**W8.** A rectangular tabletop has area 156 square inches. Its length is 1 inch more than its width. Let w be the width. Write an equation, solve by factoring, and give the physical dimensions.

**W9.** For a small fundraiser, the ticket price in dollars is 7 more than the number of tickets sold. Ticket revenue is $170. Let n be the number sold. Find n and the ticket price by factoring. If expenses are $50, what is the profit?

**W10.** A 10-meter by 6-meter garden gets a uniform walkway of width x meters around all four sides. The total outer area, including the garden, is 140 square meters. Write an equation, remove the common factor, and solve by factoring for the walkway width.