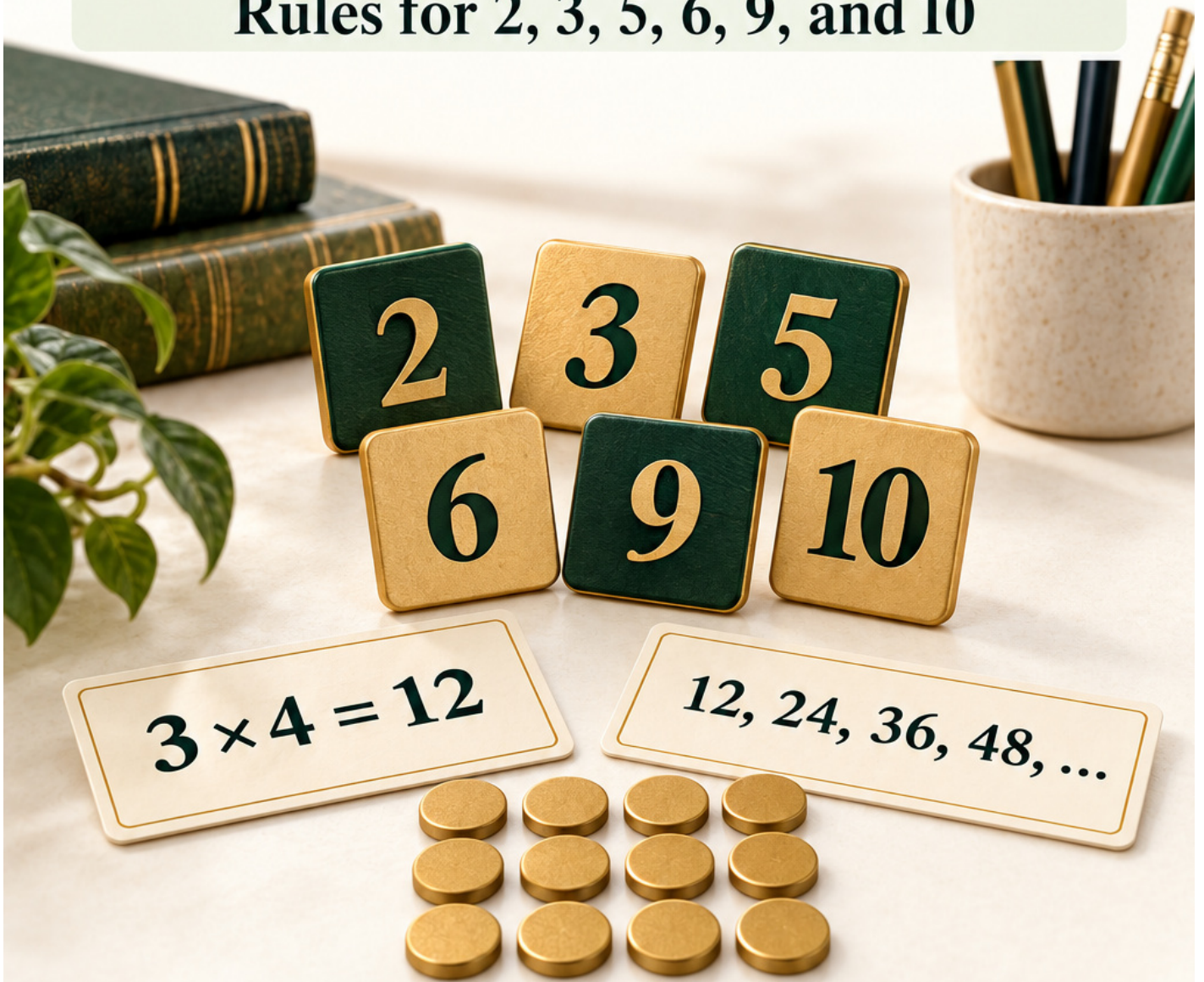


CLASS 16

Factors, Multiples, and Divisibility Rules

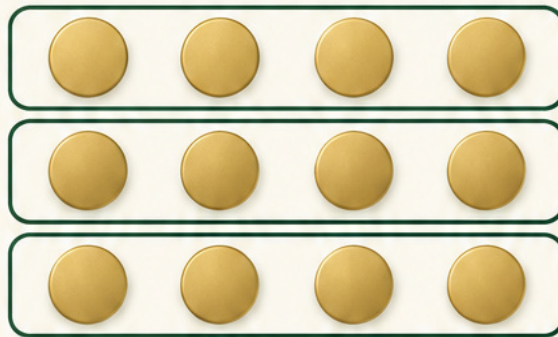
Rules for 2, 3, 5, 6, 9, and 10



Discover patterns. Make connections.

Understanding Factors

A factor divides a whole number evenly,
with no remainder.



12 counters in 3 equal groups of 4

$$12 \div 3 = 4 \quad | \quad 12 \div 4 = 3$$

3 and 4 are factors of 12.

ALL FACTORS OF 12



5 is **not** a factor of 12:
 $12 \div 5$ leaves a remainder of 2.

Try it:

Which numbers are factors of 18?

2 4 6 9

Image 1 practice: Understanding factors

Q1. Which of 3, 5, 6, and 8 are factors of 24? Explain how you know.

Q2. Is 7 a factor of 35? Write a multiplication fact to justify your answer.

Q3. Is 4 a factor of 22? Use a division statement with a remainder.

Q4. List all the positive factors of 16.

Q5. A teacher puts 28 counters into 4 equal groups with none left over. How many counters are in each group, and which factor pair does this show?

Factor Pairs

A factor pair is two whole numbers that multiply to make a given number.

Factor pairs of 12



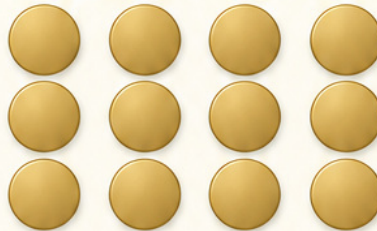
$$1 \times 12 = 12$$

(1, 12)



$$2 \times 6 = 12$$

(2, 6)



$$3 \times 4 = 12$$

(3, 4)

Find pairs systematically: start with 1 and test the next whole numbers.



Reversing a pair gives the same factors: $3 \times 4 = 4 \times 3$.

Try it:

List all the factor pairs of 18.

Image 2 practice: Finding factor pairs

Q6. List all the factor pairs of 20. Do not count a reversed pair twice.

Q7. Complete the factor pairs of 28: (1, ___), (2, ___), (4, ___).

Q8. List every factor pair of 36, including the pair with equal numbers.

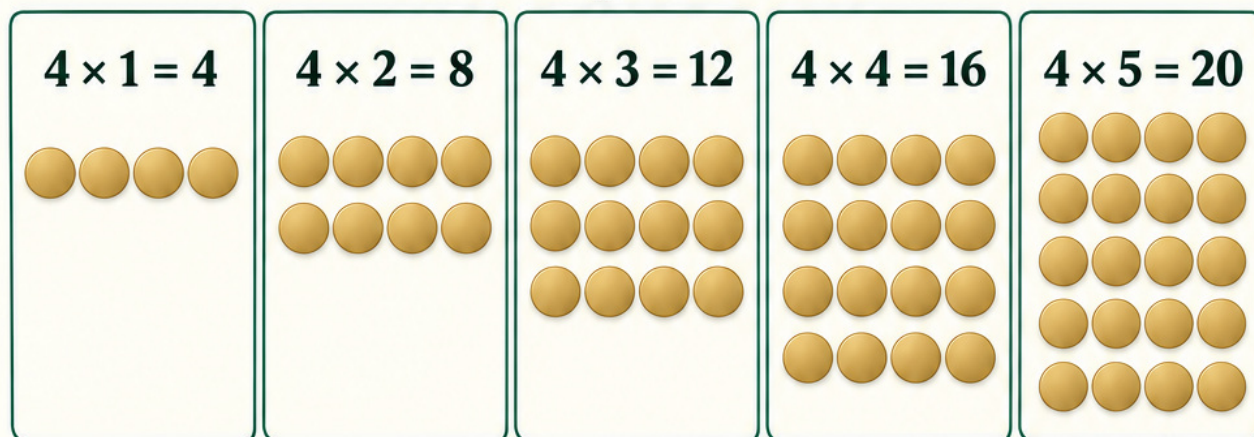
Q9. A student lists (1, 15), (3, 5), and (5, 3) as three different factor pairs of 15. Correct the list.

Q10. Square tiles cover a rectangle with an area of 32 square units. The side lengths are whole numbers. List all possible side-length pairs, ignoring rotations.

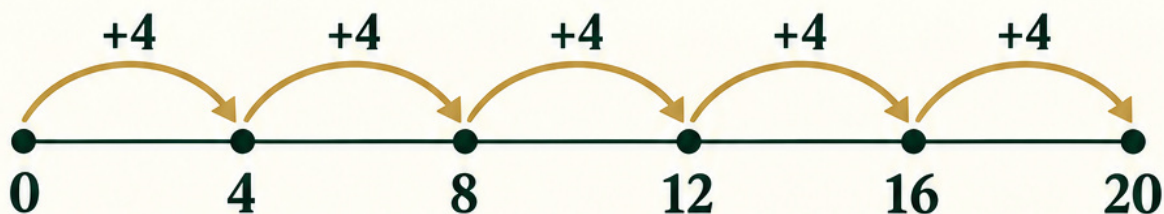
Understanding Multiples

Multiples are the results of multiplying a number by whole numbers.

Multiples of 4



Skip count by 4



0, 4, 8, 12, 16, 20, ...

The list continues forever.

Try it:

Write the first five positive multiples of 6.

Image 3 practice: Understanding multiples

Q11. Write the first five positive multiples of 7.

Q12. Continue the multiples of 8: 8, 16, 24, __, __, __.

Q13. Which of 21, 30, 41, and 54 are multiples of 6?

Q14. Is 0 a multiple of 9? Explain with an equation.

Q15. A counter starts at 0 and makes equal jumps of 5. How many jumps land exactly on 45?

Factors vs. Multiples

Two connected ideas — two different questions.

FACTORS OF 12

What divides 12 evenly?

1, 2, 3, 4, 6, 12

$$12 \div 3 = 4$$

3 is a factor of 12.

A positive whole number has
a limited number of
positive factors.

MULTIPLES OF 12

What can 12 multiply to make?

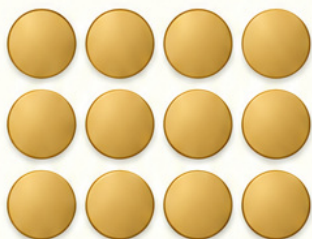
0, 12, 24, 36, 48, ...

$$12 \times 3 = 36$$

36 is a multiple of 12.

A positive whole number has
infinitely many multiples.

The connection



$$3 \times 4 = 12$$

3 and 4 are factors of 12.

12 is a multiple of both 3 and 4.

Try it:

For the number 8:

1. List all its positive factors.
2. Write its first five positive multiples.

Image 4 practice: Factors versus multiples

Q16. List all positive factors of 10 and its first five positive multiples.

Q17. Use $7 \times 6 = 42$ to complete both statements: 7 is a ___ of 42; 42 is a ___ of 7.

Q18. Relative to 15, classify each number as factor only, multiple only, both, or neither:
3, 15, 30, 8.

Q19. Correct this statement: "18 is a factor of 6 because $6 \times 3 = 18$."

Q20. Which list can continue forever: the positive factors of 7 or the positive multiples of 7? Give three positive multiples.

Divisibility by 2, 5, and 10

Look at the last digit!

Divisible means you can divide evenly, with no remainder.

DIVISIBLE BY 2

The last digit is
0, 2, 4, 6, or 8.

$$24\mathbf{8} \div 2 = 124$$

248 is divisible by 2.

DIVISIBLE BY 5

The last digit is
0 or 5.

$$13\mathbf{5} \div 5 = 27$$

135 is divisible by 5.

DIVISIBLE BY 10

The last digit is 0.

$$37\mathbf{0} \div 10 = 37$$

370 is divisible by 10.

The connection

A number ending in 0 is divisible by 2, 5, and 10.

120

$$\longrightarrow 120 \div 2 = 60$$

$$\longrightarrow 120 \div 5 = 24$$

$$\longrightarrow 120 \div 10 = 12$$

Try it:

Which of 2, 5, and 10 divide each number evenly?

46

75

90

Image 5 practice: Divisibility by 2, 5, and 10

Q21. Which of 2, 5, and 10 divide 384 evenly?

Q22. Which of 2, 5, and 10 divide 245 evenly?

Q23. Which of 2, 5, and 10 divide 610 evenly?

Q24. Lena says 405 is divisible by 10 because it is divisible by 5. Is she correct?

Q25. Replace the blank in 73_ with every possible digit that makes the number divisible by both 2 and 5.

CLASS 16 • ITEM 6

Divisibility by 3 and 9

Add the digits!

A number is divisible by 3 or 9 if the sum of its digits is divisible by 3 or 9.

DIVISIBLE BY 3

The sum of the digits
is divisible by 3.

123

$$1 + 2 + 3 = 6$$

6 is divisible by 3.

$$123 \div 3 = 41$$

DIVISIBLE BY 9

The sum of the digits
is divisible by 9.

729

$$7 + 2 + 9 = 18$$

18 is divisible by 9.

$$729 \div 9 = 81$$

The connection

Every number divisible by 9 is also divisible by 3.

729



$$729 \div 3 = 243$$

123 is divisible by 3, but not by 9.

Its digit sum is 6, which is not divisible by 9.

Try it:

Is each number divisible by 3, by 9, by both, or by neither?

234

513

421

Image 6 practice: Divisibility by 3 and 9

Q26. Is 312 divisible by 3, by 9, by both, or by neither? Show the digit sum.

Q27. Is 459 divisible by 3, by 9, by both, or by neither?

Q28. Is 527 divisible by 3 or 9? Explain without long division.

Q29. Find every digit that can replace the blank in 4_2 to make a number divisible by 3.

Q30. A student claims every number divisible by 3 is divisible by 9. Use 114 to test the claim.

CLASS 16 • ITEM 7

Divisibility by 6

Check BOTH 2 and 3.

A number is divisible by 6 if it is divisible by both 2 and 3.

246

STEP 1: CHECK 2

Last digit: 6

246 is even.



Divisible by 2.

STEP 2: CHECK 3

$$2 + 4 + 6 = 12$$

12 is divisible by 3.



Divisible by 3.

Both checks pass!

$$246 \div 6 = 41$$

One check is not enough

124

Even: yes



Digit sum: $1 + 2 + 4 = 7$



Not divisible by 6.

123

Digit sum: $1 + 2 + 3 = 6$



Even: no



Not divisible by 6.

Try it:

Which numbers are divisible by 6?

132

215

324

Image 7 practice: Divisibility by 6: both tests must pass

Q31. Is 138 divisible by 6? Show both required checks.

Q32. Is 214 divisible by 6? Explain which check fails.

Q33. Is 357 divisible by 6? Explain which check fails.

Q34. Which of 144, 250, 333, and 402 are divisible by 6?

Q35. Find every digit that can replace the blank in 25_ to make a number divisible by 6.

Applying Divisibility Rules

Group, share, and solve number puzzles.

1. GROUPING BOOKS

Can 126 books be packed into boxes of 6 with none left over?



Last digit 6 → divisible by 2.



$1 + 2 + 6 = 9$ → divisible by 3.



Yes! $126 \div 6 = 21$ boxes.

2. SHARING PENCILS

Can 135 pencils be shared equally among 9 students?



$1 + 3 + 5 = 9$ → divisible by 9.



Yes! $135 \div 9 = 15$ pencils each.

3. NUMBER PUZZLE

I am between 40 and 50. I am divisible by both 5 and 9. What number am I?



Multiples of 5 between 40 and 50: 45



$4 + 5 = 9$ → divisible by 9.



The number is 45.

Try it:

A museum has 150 tickets. Can they be bundled in groups of 2, 3, 5, 6, 9, or 10 with none left over? Explain using the rules.



Image 8 practice: Applying divisibility rules

Q36. Can 168 pencils be packed into boxes of 6 with none left over? If so, how many boxes are needed?

Q37. Can 216 counters be shared equally among 9 children with none left over? How many does each child get?

Q38. A shop has 235 stickers. Can it make full packs of 5? Can it make full packs of 10 with none left over?

Q39. A whole number is greater than 80 and less than 100. It is divisible by both 3 and 10. What is the number?

Q40. A museum has 270 tickets. Which group sizes from 2, 3, 5, 6, 9, and 10 use all tickets with none left over?

Homework review: R1–R5

R1. List all positive factors of 30.

R2. List every factor pair of 40, without counting reversals twice.

R3. Write the first five positive multiples of 9.

R4. Use $8 \times 7 = 56$ to write two factor statements and two multiple statements.

R5. Which of 2, 5, and 10 divide 1,250 evenly?

Homework review: R6–R8

R6. Which of 3 and 9 divide 642 evenly?

R7. Is 426 divisible by 6? Show both checks.

R8. Find every whole number greater than 100 and less than 130 that is divisible by both 5 and 6.

Homework word problems: W1–W5

W1. A gardener has 42 seedlings. Can she put exactly 6 seedlings in each tray with none left over? How many trays would she fill?

W2. A craft club has 32 beads. Which of 2, 3, 5, or 8 children could share all the beads equally with none left over? Give the share for each possible group.

W3. A teacher arranges 48 chairs in equal rows. List every pair of row counts and chairs per row, ignoring reversed arrangements.

W4. Each packet holds 7 seeds. Write the total numbers of seeds in 1, 2, 3, and 4 full packets. Is 31 seeds a possible total using only full packets?

W5. A school has 284 gloves. Can they all be made into pairs? How many pairs are there?

Homework word problems: W6–W10

W6. A baker has 365 cookies and uses bags holding exactly 5 cookies. Can all cookies be packed with no leftovers? How many bags are needed?

W7. A shop packs 470 cards into bundles of 10. How many complete bundles can it make, and will any cards remain?

W8. Three classes share 315 craft sticks equally. How many sticks does each class get? Could 9 groups also share that total equally?

W9. A museum has 208 tickets and wants packs of 6. Can it pack every ticket with none left over? If not, how many full packs and leftover tickets are there?

W10. A club has 1,134 stickers for 9 equal prize bags. Use a divisibility rule, then find the number in each bag.

Homework word problems: W11–W12

W11. A box label says 5_4 marbles. The box can be shared equally among 6 students with no marbles left. Find every possible missing digit.

W12. A teacher has more than 150 but fewer than 200 counters. They can be put into groups of 10 or into groups of 9 with none left over. How many counters are there, and how many groups would each arrangement make?