SCHOOL OF MATH

CLASS 20

Multiplying Fractions

By Whole Numbers and by Fractions

School of Math New York | SOMATH
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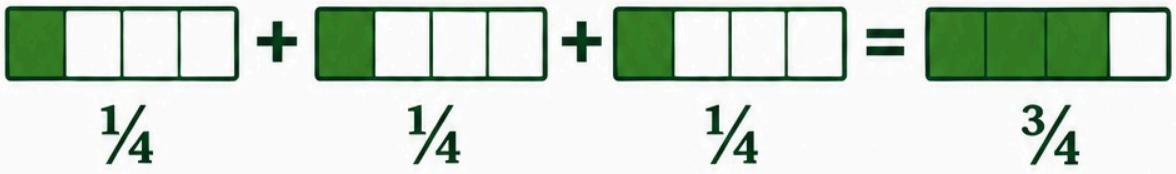
SCHOOL OF MATH | CLASS 20, ITEM 1

Multiplication as Repeated Addition

Equal groups of a fraction

$$3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

3 groups of one-fourth make three-fourths.



$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{3}{4}$

EXAMPLE

Each ribbon is $\frac{1}{5}$ meter long. What is the total length of 4 ribbons?

$\frac{1}{5}$ m | $\frac{1}{5}$ m | $\frac{1}{5}$ m | $\frac{1}{5}$ m

$$4 \times \frac{1}{5} = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{4}{5} \text{ meter}$$

Multiply to add equal fractions quickly!

Multiplication as repeated addition

Q1. Write $5 \times \frac{1}{6}$ as a repeated addition. Then find the product.

Q2. Write $\frac{2}{9} + \frac{2}{9} + \frac{2}{9} + \frac{2}{9}$ as a multiplication. Then find the total.

Q3. A small glass holds $\frac{1}{8}$ liter of juice. How much juice is in 7 glasses?

Q4. Is $6 \times \frac{1}{6}$ more than, less than, or equal to 1 whole? Explain using equal groups.

Q5. One lap around a small track is $\frac{1}{10}$ mile. Mia runs 9 laps. Write a repeated addition and a multiplication, then find how far she runs.

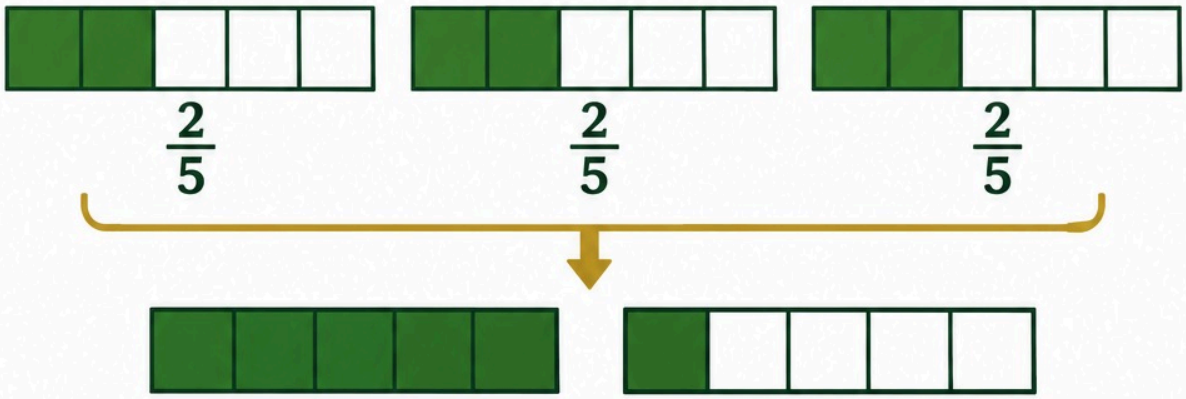
SCHOOL OF MATH | CLASS 20 | ITEM 2

Multiplying a Fraction by a Whole Number

Multiply the numerator. Keep the denominator.

$$3 \times \frac{2}{5} = \frac{3 \times 2}{5} = \frac{6}{5} = 1\frac{1}{5}$$

1. Multiply the whole number by the numerator.
2. Keep the same denominator.
3. Simplify or write as a mixed number.



6 fifths = 1 whole and 1 fifth

Another example $4 \times \frac{3}{8} = \frac{12}{8} = \frac{3}{2} = 1\frac{1}{2}$

The denominator tells us the size of each piece.

Multiplying a fraction by a whole number

Q6. Find $4 \times \frac{2}{9}$.

Q7. Find $5 \times \frac{3}{7}$. Write the answer as a mixed number.

Q8. Find $6 \times \frac{5}{8}$. Simplify and write the answer as a mixed number.

Q9. Find $3 \times \frac{4}{5}$. Write the answer as a mixed number.

Q10. Ben says $2 \times \frac{3}{5} = \frac{6}{10}$. Explain his mistake and find the correct product.

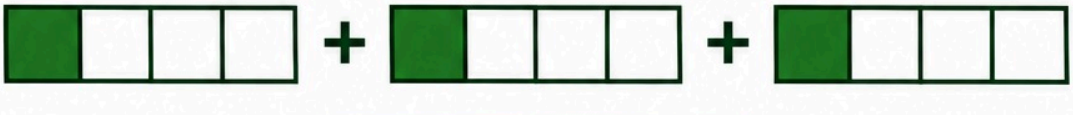
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CLASS 20 ITEM 3

Using Visual Models

See why 3 × ¼ = ¾

Fraction Strips





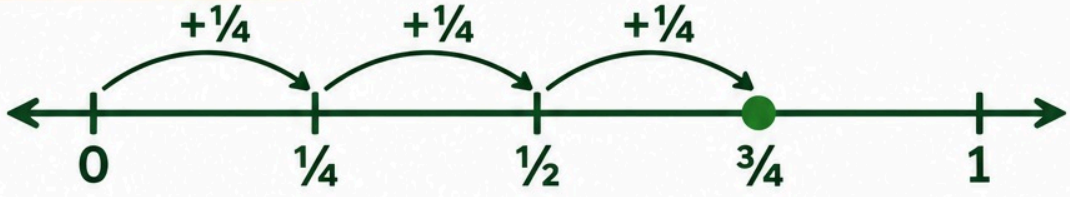
$$\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$$

Shaded Shapes



Three one-fourth pieces make three-fourths.

Number Line



Different models. The same product: ¾.

Using visual models

Q11. The model shows equal strips. Each strip is cut into sixths. Write the multiplication that the model shows, then find the product.



Q12. The number line from 0 to 1 is cut into eighths. Start at 0 and make 5 jumps of $\frac{1}{8}$. Where do you land? Write the multiplication.



Q13. A circle is cut into 6 equal slices. You shade $\frac{1}{6}$ of the circle five times, one slice at a time. What fraction of the circle is shaded, and what fraction is not shaded?

Q14. Use the number line from 0 to 2, cut into fifths, to show $3 \times \frac{2}{5}$. How long is each jump, and where do you land?



Q15. Draw strips to show $4 \times \frac{1}{3}$. How many whole strips can you fill, and what is the product?

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CLASS 20 • ITEM 4

Multiplying Fractions by Fractions

Multiply across the top and across the bottom.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Denominators must be nonzero.

1. Multiply the numerators.

2. Multiply the denominators.

$$\frac{2}{3} \times \frac{3}{5} = \frac{2 \times 3}{3 \times 5} = \frac{6}{15} = \frac{2}{5}$$

Numerators: $2 \times 3 = 6$

Denominators: $3 \times 5 = 15$

Simplify: $\frac{6}{15} = \frac{2}{5}$

Try another example

$$\frac{1}{4} \times \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$$

Your turn:

$$\frac{3}{4} \times \frac{2}{5} = ?$$

Multiply first. Then simplify your answer.

Multiplying fractions by fractions

Q16. Find $\frac{1}{3} \times \frac{4}{5}$.

Q17. Find $\frac{2}{7} \times \frac{3}{4}$. Simplify your answer.

Q18. Find $\frac{5}{6} \times \frac{1}{2}$.

Q19. Find $\frac{3}{5} \times \frac{5}{8}$. Simplify your answer.

Q20. Ana writes $\frac{2}{5} \times \frac{1}{3} = \frac{3}{8}$. What did she do wrong? Find the correct product.

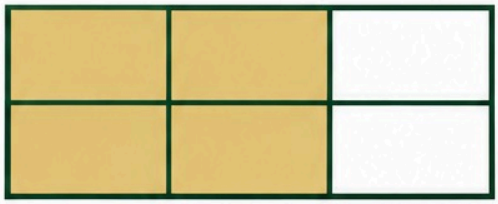
SCHOOL OF MATH | CLASS 20 • ITEM 5

Finding a Fraction of a Fraction

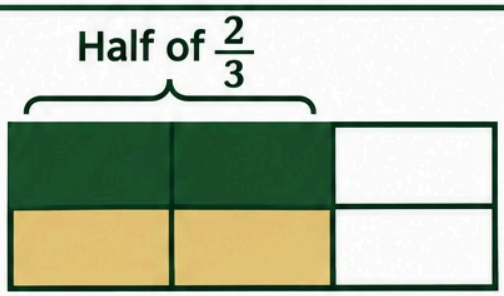
"Of" means multiply.

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

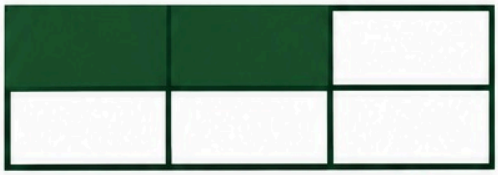
1. Start with two-thirds.



2. Take half of the shaded part.

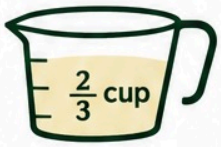


3. Compare with the whole.



2 of 6 equal parts = $\frac{2}{6} = \frac{1}{3}$

REAL-WORLD EXAMPLE



A recipe needs $\frac{2}{3}$ cup of milk. For half the recipe, use $\frac{1}{3}$ cup.

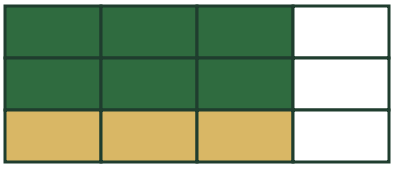
Half of two-thirds is one-third.

Finding a fraction of a fraction

Q21. Find $\frac{1}{2}$ of $\frac{3}{4}$.

Q22. Find $\frac{1}{3}$ of $\frac{3}{5}$. Simplify your answer.

Q23. A rectangle is cut into 4 equal columns, and 3 columns are lightly shaded to show $\frac{3}{4}$. It is also cut into 3 equal rows, and 2 rows of the shaded part are shaded dark. What fraction of the whole rectangle is dark? Write the multiplication.



Q24. A garden is $\frac{4}{5}$ planted. Carrots grow in $\frac{1}{4}$ of the planted part. What fraction of the whole garden has carrots?

Q25. There is $\frac{3}{4}$ of a tray of brownies left. Sam eats $\frac{1}{3}$ of what is left. What fraction of the whole tray does Sam eat?

SCHOOL OF MATH | CLASS 20 • ITEM 6

Simplifying Products

Smaller numbers. The same answer.

Method 1: Multiply, then simplify

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

Divide 6 and 12 by their common factor, 6.

Method 2: Simplify before multiplying

$$\frac{2}{3} \times \frac{3}{4}$$

1. Divide the numerator 3 and denominator 3 by 3.
2. Divide the numerator 2 and denominator 4 by 2.

$$\frac{1}{1} \times \frac{1}{2} = \frac{1}{2}$$

Rule: Simplify a numerator and a denominator by the same common factor.

With a whole number

$$6 \times \frac{5}{9} = \frac{6}{1} \times \frac{5}{9} = \frac{2}{1} \times \frac{5}{3} = \frac{10}{3} = 3\frac{1}{3}$$

Your turn: $\frac{3}{8} \times \frac{4}{9} = ?$

Simplifying products two ways

Q26. Find $\frac{4}{9} \times \frac{3}{8}$ two ways: multiply then simplify, and simplify before multiplying.

Q27. Simplify before multiplying: $\frac{5}{6} \times \frac{3}{10}$.

Q28. Find $8 \times \frac{3}{4}$ by simplifying first.

Q29. Find $10 \times \frac{7}{15}$. Simplify first and write the answer as a mixed number.

Q30. Simplify before multiplying: $\frac{6}{7} \times \frac{7}{12}$.

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CLASS 20 • ITEM 7

Understanding the Size of the Product

Predict before you multiply!

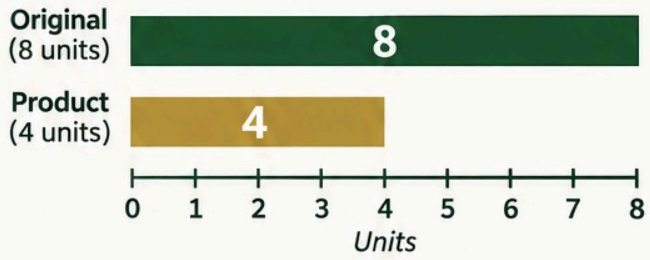
Start with a positive number.

Multiply by a number between 0 and 1

The product is smaller than the starting number.

$$8 \times \frac{1}{2} = 4$$

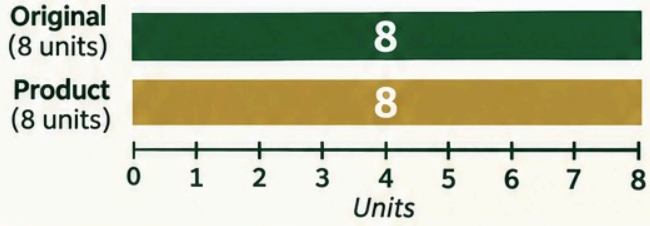
$4 < 8$



Multiply by 1

The product stays the same.

$$8 \times 1 = 8$$

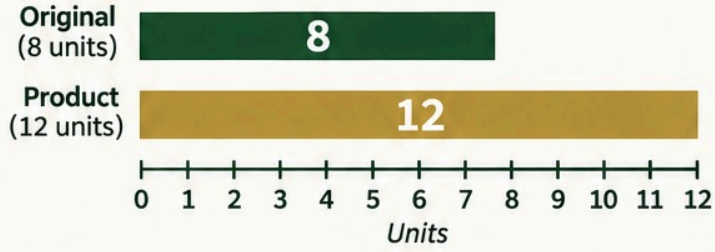


Multiply by a number greater than 1

The product is larger than the starting number.

$$8 \times \frac{3}{2} = 12$$

$12 > 8$



Think first: Is $\frac{3}{4} \times \frac{1}{2}$ greater or less than $\frac{3}{4}$?

Less: *taking half makes it smaller.*

Understanding the size of the product

Q31. Without multiplying, decide whether $9 \times \frac{2}{3}$ is greater than or less than 9. Then find the product to check.

Q32. Is $6 \times \frac{5}{4}$ greater than or less than 6? Explain, then find the product as a mixed number.

Q33. Order from least to greatest without finding exact answers: $10 \times \frac{7}{5}$, 10×1, $10 \times \frac{3}{5}$. Then check by multiplying.

Q34. Is $\frac{2}{3} \times \frac{4}{5}$ greater than or less than $\frac{4}{5}$? Explain, then check.

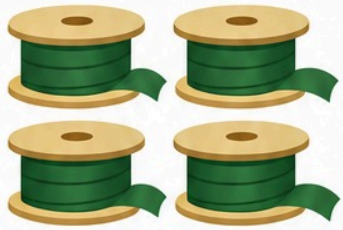
Q35. Write <, >, or = in each blank: (a) $12 \times \frac{5}{5}$ ___ 12 (b) $12 \times \frac{4}{3}$ ___ 12

SCHOOL OF MATH | CLASS 20 • ITEM 8

Solving Real-World Problems

Read • Multiply • Simplify • Label

Equal Groups



Each ribbon is $\frac{3}{4}$ meter long.
How much ribbon do 4 pieces use?

$$4 \times \frac{3}{4} = \frac{12}{4} = 3$$

Total: 3 meters of ribbon.

Recipes

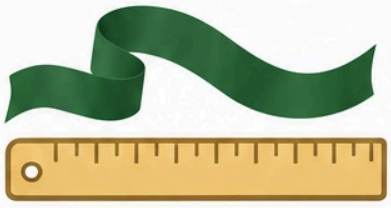


A recipe needs $\frac{2}{3}$ cup of milk.
How much milk is needed for half the recipe?

$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$$

Use $\frac{1}{3}$ cup of milk.

Measurements



A ribbon is $\frac{5}{6}$ meter long.
You use $\frac{3}{5}$ of it. How much do you use?

$$\frac{3}{5} \times \frac{5}{6} = \frac{15}{30} = \frac{1}{2}$$

You use $\frac{1}{2}$ meter of ribbon.

Always include the unit in your answer!

Solving real-world fraction problems

Q36. Each bag holds $\frac{2}{3}$ pound of trail mix. How much trail mix is in 5 bags?

Q37. A recipe needs $\frac{3}{4}$ cup of sugar. Maya makes one-third of the recipe. How much sugar does she use?

Q38. A wooden board is $\frac{7}{8}$ meter long. Theo uses $\frac{2}{3}$ of it for a shelf. How long is the shelf?

Q39. There are 24 students in a class, and $\frac{3}{8}$ of them walk to school. How many students walk to school?

Q40. A painter uses $\frac{5}{6}$ gallon of paint for each wall. How much paint does she need for 4 walls?

Homework review

R1. Write $\frac{3}{7} + \frac{3}{7} + \frac{3}{7}$ as a multiplication and find the total as a mixed number.

R2. Find $7 \times \frac{3}{10}$. Write the answer as a mixed number.

R3. Use the number line from 0 to 2, cut into thirds, to show $4 \times \frac{1}{3}$. Where do you land?



R4. Find $\frac{4}{9} \times \frac{2}{3}$.

R5. Find $\frac{3}{4}$ of $\frac{2}{5}$. Simplify your answer.

Homework review (continued)

R6. Simplify before multiplying: $\frac{9}{10} \times \frac{5}{12}$.

R7. Without multiplying, is $15 \times \frac{3}{5}$ greater than or less than 15? Then find the product.

R8. Each bottle holds $\frac{3}{5}$ liter of water. How much water is in 6 bottles?

Homework word problems

W1. Each of 3 children gets $\frac{1}{4}$ of a sandwich. How much sandwich do they get in all?

W2. A puppy eats $\frac{2}{5}$ cup of food at each meal and has 3 meals a day. How much food does it eat in one day?

W3. A ribbon is $\frac{5}{6}$ yard long. Maya uses $\frac{1}{2}$ of it for a bow. How much ribbon is in the bow?

W4. Leo walks $\frac{3}{4}$ mile to school each morning. How far does he walk to school in 5 mornings?

W5. In a club, $\frac{2}{3}$ of the members play chess. Half of the chess players are in 4th grade. What fraction of the whole club are 4th-grade chess players?

Homework word problems (continued)

W6. A pitcher holds $\frac{9}{10}$ liter of lemonade. Kai pours out $\frac{2}{3}$ of it into cups. How much lemonade does he pour?

W7. One batch of muffins needs $\frac{3}{4}$ cup of flour. How much flour is needed for 6 batches?

W8. A jar has 30 marbles, and $\frac{2}{5}$ of them are blue. How many marbles are blue?

W9. A garden covers $\frac{5}{8}$ of a yard. Tomatoes grow in $\frac{2}{5}$ of the garden. What fraction of the whole yard has tomatoes? Is that more or less than $\frac{5}{8}$?

W10. Zoe has $\frac{4}{5}$ of a pizza left. She gives $\frac{1}{2}$ of the leftover pizza to her brother and eats $\frac{1}{4}$ of the leftover pizza herself. What fraction of the whole pizza is still left?

Homework word problems (continued)

W11. A plant is 2 inches tall. It grows $\frac{3}{4}$ inch every week. How tall is it after 6 weeks?

W12. A rug has an area of 12 square feet. $\frac{2}{3}$ of the rug is red, and $\frac{3}{8}$ of the red part has a star pattern. How many square feet have the star pattern?