SOMATH — SCHOOL OF MATH

Pre-Algebra Class: 6 Foundational Concepts

Whole Numbers → Place Value → Operations → Properties → Factors & Multiples → Integers → Equivalent Fractions

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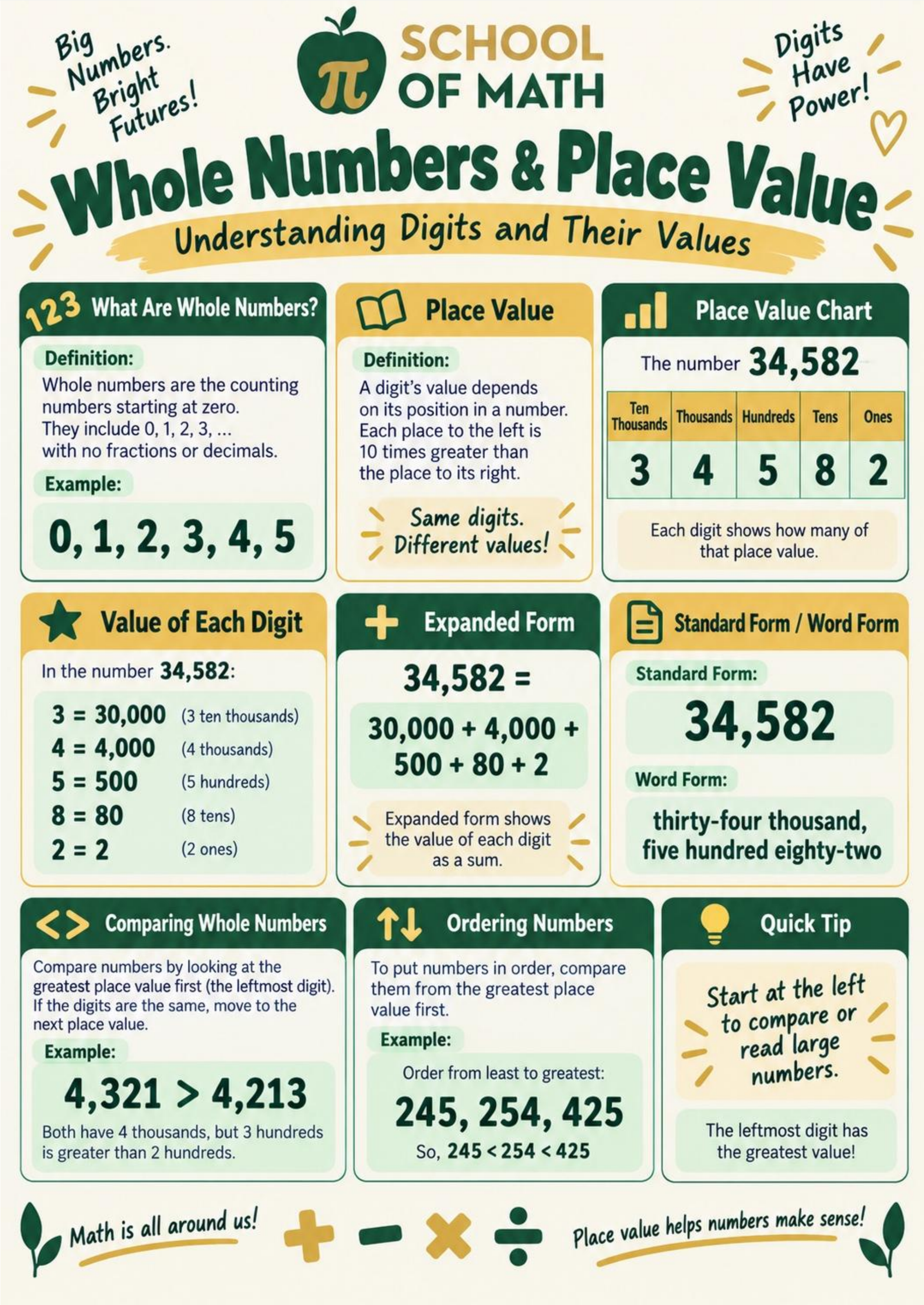
Class Plan

1. Whole Numbers & Place Value
2. Operations with Whole Numbers
3. Number Properties
4. Factors and Multiples
5. Integers
6. Equivalent Fractions

1. Whole Numbers & Place Value



What are whole numbers?

The **whole numbers** are the counting numbers starting at zero: `0, 1, 2, 3, 4, 5, …`. Whole numbers have **no** fractions, decimals, or negatives.

Place value

The **place value** of a digit depends on **where** the digit sits in the number. Each place to the left is **10 times greater** than the place to its right.

> *Same digits — different values. In **522**, the two 2s do not mean the same thing: one is **2 tens** (20), the other is **2 ones** (2).*

Place value chart (for 34,582)

Ten Thousands	Thousands	Hundreds	Tens	Ones
3	4	5	8	2

Value of each digit in 34,582

- 3 → 30,000 (3 ten thousands)
- 4 → 4,000 (4 thousands)
- 5 → 500 (5 hundreds)
- 8 → 80 (8 tens)
- 2 → 2 (2 ones)

Standard form, expanded form, word form

- **Standard form:** 34,582
- **Expanded form:** 30,000 + 4,000 + 500 + 80 + 2
- **Word form:** thirty-four thousand, five hundred eighty-two

Comparing and ordering whole numbers

Compare from the **leftmost** place (the largest place value) first. If those digits match, move one place right and compare again.

Example: 4,321 > 4,213 — both have 4 thousands, but 3 hundreds > 2 hundreds.

To order least → greatest, always compare from the greatest place value first. Example: 245 < 254 < 425.

Quick tip: Start at the left to compare or read large numbers — the leftmost digit carries the greatest value.

10 Practice Questions

Q1. What is the value of the digit 7 in **27,304**?

Q2. Write **58,206** in expanded form.

Q3. Write **forty thousand, three hundred nine** in standard form.

Q4. Which number is greater: **63,481** or **63,418**?

Q5. Order from least to greatest: **5,082** **5,802** **5,028** **5,820**.

Q6. How many hundreds are in **12,347**?

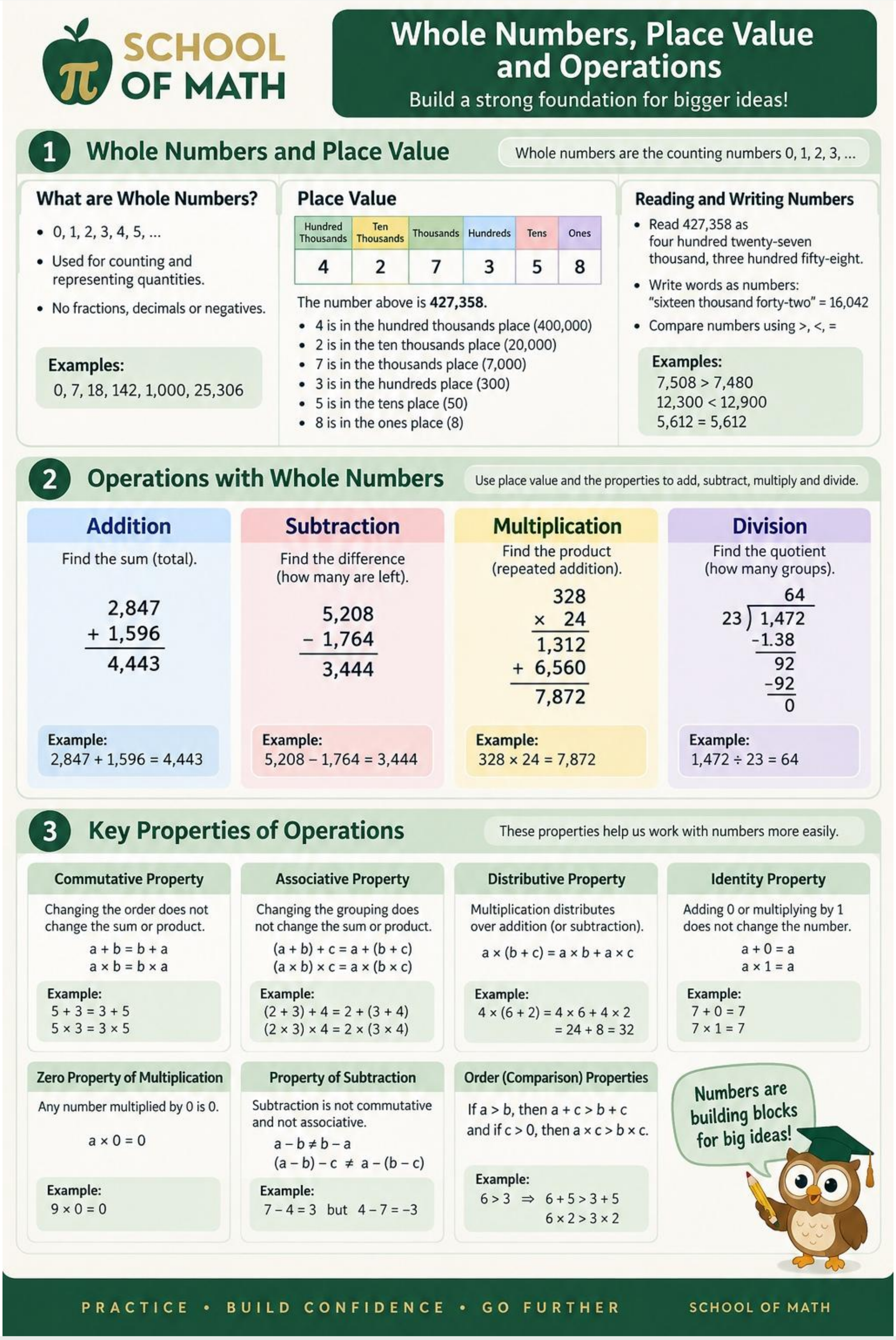
Q7. What is the place value of the underlined digit in 4<u>**5**</u>,678?

Q8. Write 6,000 + 300 + 40 + 2 in standard form.

Q9. Which is the greatest whole number less than 10,000?

Q10. A digit’s value moves one place to the LEFT. What happens to its value?

2. Operations with Whole Numbers



The four operations, one place-value system

Whole-number arithmetic uses the same idea over and over: **line up place values**, operate one column at a time, and carry / borrow when a place overflows or falls short. Every algorithm below is just place value + the properties of operations.

Addition — find the sum (total)

Line up ones under ones, tens under tens, hundreds under hundreds. Add each column right to left. If a column sum is 10 or more, **carry** 1 to the next place.

Example: 2,847 + 1,596 = **4,443**.

Subtraction — find the difference (how many are left)

Line up place values. Subtract right to left. If the top digit is smaller than the bottom digit, **borrow** 1 from the next place to the left (which is worth 10 in your current place).

Example: 5,208 − 1,764 = **3,444**.

Multiplication — find the product (repeated addition)

Multiplying a number by `n` is adding it to itself `n` times. In multi-digit multiplication, multiply by each digit of the bottom number (ones, then tens, then hundreds), shifting each partial product one place left, and add.

Example: 328 × 24. First 328 × 4 = 1,312. Then 328 × 2 (tens) = 656 → written as 6,560. Sum: 1,312 + 6,560 = **7,872**.

Division — find the quotient (how many groups)

Division asks: how many times does the divisor fit into the dividend? Use long division: divide, multiply, subtract, bring down.

Example: 1,472 ÷ 23 = **64**.

Reading and writing numbers

Read 427,358 as “four hundred twenty-seven thousand, three hundred fifty-eight.” Write “sixteen thousand forty-two” as 16,042.

10 Practice Questions

Q1. Compute 4,687 + 2,548.

Q2. Compute 8,003 − 2,547.

Q3. Compute 236 × 47.

Q4. Compute 1,296 ÷ 8.

Q5. Estimate 4,872 + 3,158 by rounding each to the nearest thousand.

Q6. A theater has 24 rows of 38 seats. How many seats total?

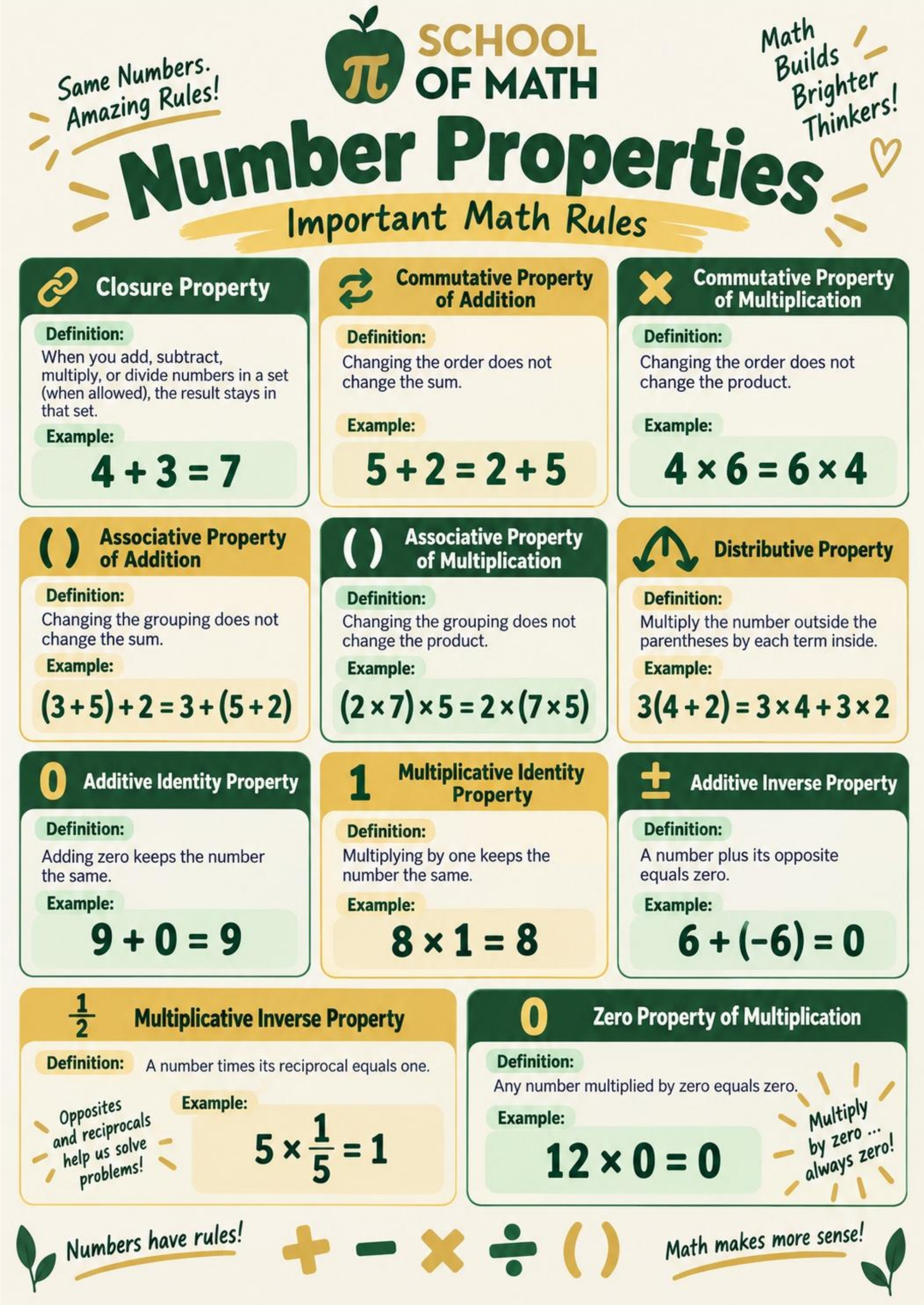
Q7. Divide 4,725 by 15.

Q8. Write the number 305,072 in words.

Q9. Compare using >, <, or =: 9,876 ? 9,867.

Q10. A truck holds 1,250 kg. If 6 identical crates weigh 7,320 kg total, how much does one crate weigh, and does one crate fit?

3. Number Properties



Number properties = shortcuts you already use

These "rules" describe how **+, −, ×, ÷** actually behave. Knowing them by name lets you rearrange expressions confidently, do mental math, and later solve equations with variables.

Closure

If you add, subtract, multiply, or divide numbers **in a set** (when allowed), the result stays in that set. Example: any whole number + any whole number = whole number.

Commutative Property

Order does not matter for addition and multiplication.

- Addition: `a + b = b + a` → 5 + 2 = 2 + 5
- Multiplication: `a × b = b × a` → 4 × 6 = 6 × 4

Subtraction and division are NOT commutative: 7 − 4 ≠ 4 − 7.

Associative Property

Grouping does not matter for addition and multiplication.

- (3 + 5) + 2 = 3 + (5 + 2)
- (2 × 7) × 5 = 2 × (7 × 5)

Subtraction and division are NOT associative: (a − b) − c ≠ a − (b − c).

Distributive Property

Multiplication distributes over addition (or subtraction): `a × (b + c) = a×b + a×c`. Example: 3(4 + 2) = 3·4 + 3·2 = 12 + 6 = 18.

Identity Properties

- **Additive identity:** a + 0 = a → adding zero keeps the number.
- **Multiplicative identity:** a × 1 = a → multiplying by one keeps the number.

Inverse Properties

- **Additive inverse:** a + (−a) = 0. The opposite of 6 is −6.
- **Multiplicative inverse (reciprocal):** a × (1/a) = 1 (for a ≠ 0). Example: 5 × 1/5 = 1.

Zero Property of Multiplication

Any number times zero equals zero: $a \times 0 = 0$. Example: $12 \times 0 = 0$.

Order (comparison) properties

If $a > b$, then $a + c > b + c$. If also $c > 0$, then $a \times c > b \times c$. (Multiplying an inequality by a positive number preserves the direction; multiplying by a negative flips it — you will meet that later.)

10 Practice Questions

Q1. Name the property: $6 + 9 = 9 + 6$.

Q2. Name the property: $(2 \times 5) \times 4 = 2 \times (5 \times 4)$.

Q3. Fill the blank using the distributive property: $7(3 + 5) = 7 \cdot 3 +$ _____.

Q4. What is 143×1?

Q5. What is $89 + 0$?

Q6. What is the additive inverse of -12?

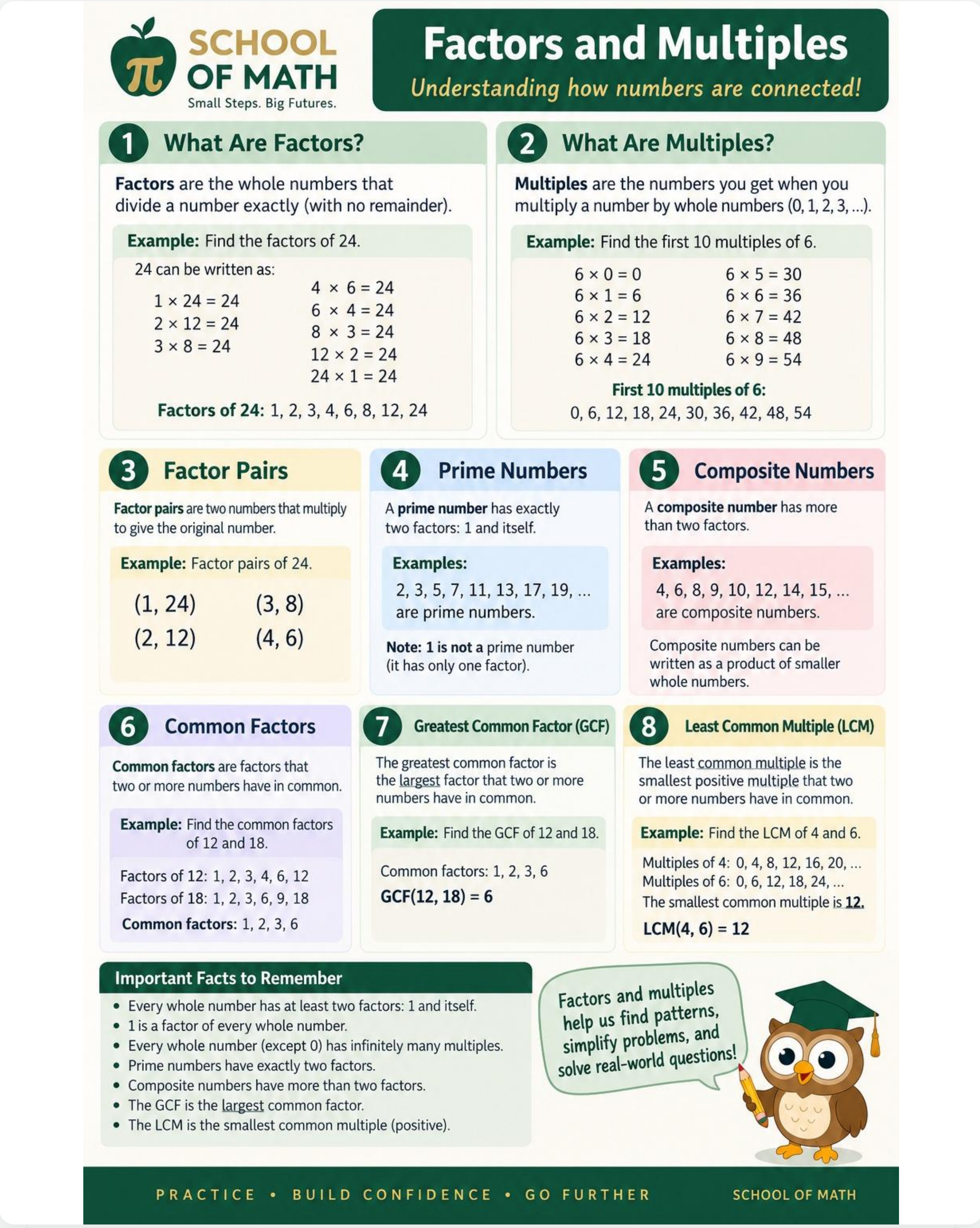
Q7. What is the multiplicative inverse (reciprocal) of 4?

Q8. Use the distributive property to compute 6×27 mentally.

Q9. True or false: subtraction is commutative.

Q10. Simplify using the zero property: $15 \times 0 + 8$.

4. Factors and Multiples



Factors

Factors of a number are the whole numbers that divide it exactly (no remainder).

Example — find the factors of 24. Write every product of two whole numbers that equals 24: 1×24, 2×12, 3×8, 4×6. So the factors of 24 are **1, 2, 3, 4, 6, 8, 12, 24**.

Multiples

Multiples of a number are what you get by multiplying it by 0, 1, 2, 3, ...

Example — first 10 multiples of 6: 0, 6, 12, 18, 24, 30, 36, 42, 48, 54.

Factor pairs

Factor pairs are two whole numbers whose product is the original number. Factor pairs of 24: (1, 24), (2, 12), (3, 8), (4, 6).

Prime and composite numbers

- **Prime:** exactly two factors — 1 and itself. Examples: 2, 3, 5, 7, 11, 13, 17, 19, ...
- **Composite:** more than two factors. Examples: 4, 6, 8, 9, 10, 12, 14, 15, ...
- **1 is not prime** (it has only one factor). 0 is neither prime nor composite.

Common factors

Factors that **two or more** numbers share. Example: factors of 12 = 1, 2, 3, 4, 6, 12; factors of 18 = 1, 2, 3, 6, 9, 18. Common factors = **1, 2, 3, 6**.

Greatest Common Factor (GCF)

The **largest** factor two or more numbers share. GCF(12, 18) = **6**. Use it to simplify fractions and factor expressions.

Least Common Multiple (LCM)

The **smallest positive** multiple two or more numbers share. Example: multiples of 4 = 0, 4, 8, 12, 16, 20, ...; multiples of 6 = 0, 6, 12, 18, 24, ... LCM(4, 6) = **12**. Use it to add fractions with unlike denominators.

> **Facts to remember.** Every whole number has at least two factors (1 and itself, except 0 and 1). 1 is a factor of every whole number. Every whole number (except 0) has infinitely many multiples.

10 Practice Questions

Q1. List all factors of 36.

Q2. List the first 8 multiples of 7 (starting at 0).

Q3. Give all factor pairs of 30.

Q4. Is 29 prime or composite? Justify.

Q5. Is 51 prime or composite?

Q6. Find GCF(24, 36).

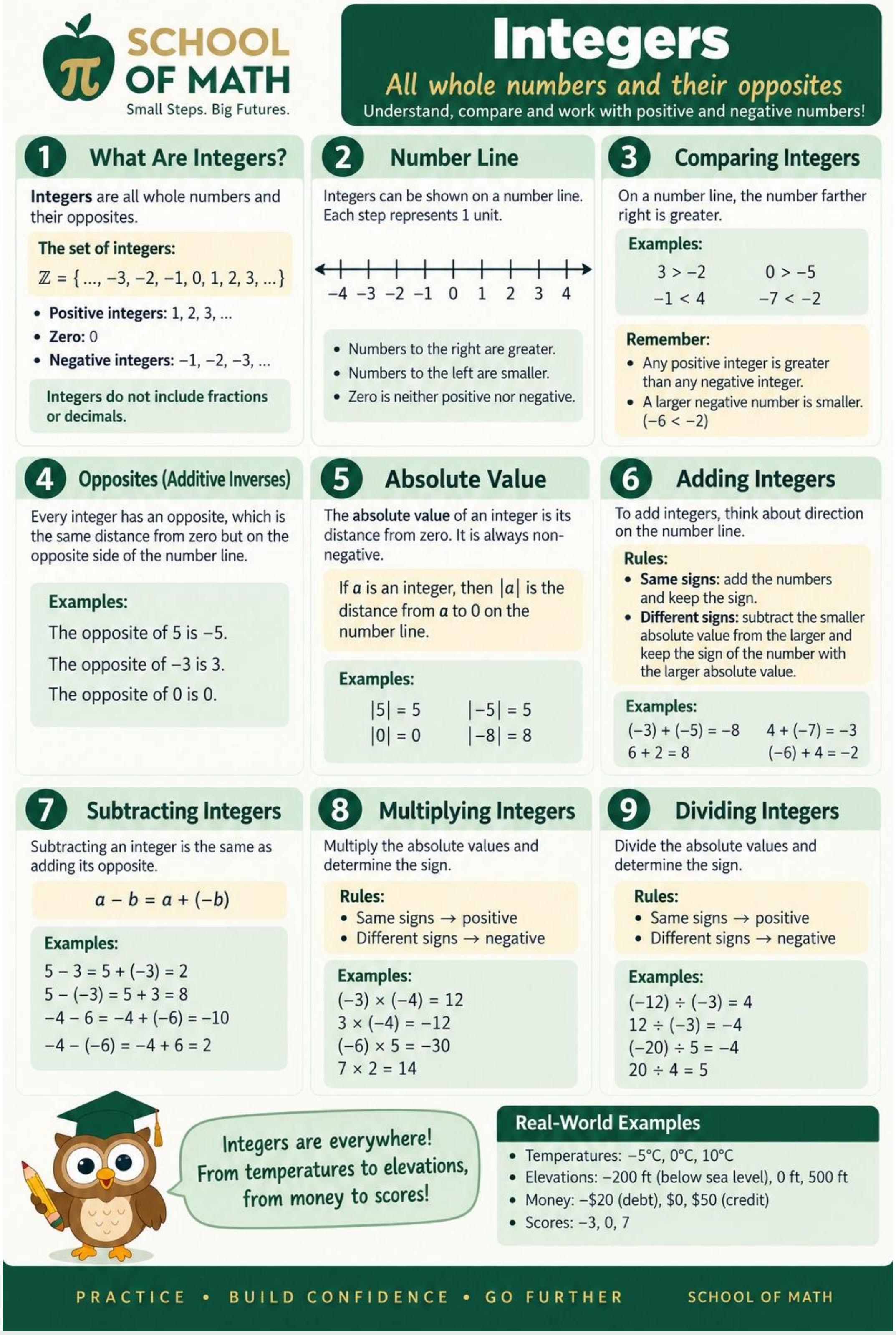
Q7. Find LCM(8, 12).

Q8. Find the common factors of 20 and 30.

Q9. Find GCF(45, 60) and LCM(45, 60).

Q10. Two lights blink every 6 seconds and 9 seconds. If they blink together at time 0, when do they next blink together?

5. Integers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Small Steps. Big Futures.
Integers
All whole numbers and their opposites
Understand, compare and work with positive and negative numbers!
1 What Are Integers?
Integers are all whole numbers and their opposites.
The set of integers:
ℤ = {..., −3, −2, −1, 0, 1, 2, 3, ...}
• Positive integers: 1, 2, 3, ...
• Zero: 0
• Negative integers: −1, −2, −3, ...
Integers do not include fractions or decimals.
2 Number Line
Integers can be shown on a number line. Each step represents 1 unit.
−4 −3 −2 −1 0 1 2 3 4
• Numbers to the right are greater.
• Numbers to the left are smaller.
• Zero is neither positive nor negative.
3 Comparing Integers
On a number line, the number farther right is greater.
Examples:
3 > −2
0 > −5
−1 < 4
−7 < −2
Remember:
• Any positive integer is greater than any negative integer.
• A larger negative number is smaller. (−6 < −2)
4 Opposites (Additive Inverses)
Every integer has an opposite, which is the same distance from zero but on the opposite side of the number line.
Examples:
The opposite of 5 is −5.
The opposite of −3 is 3.
The opposite of 0 is 0.
5 Absolute Value
The absolute value of an integer is its distance from zero. It is always non-negative.
If a is an integer, then |a| is the distance from a to 0 on the number line.
Examples:
|5| = 5
|−5| = 5
|0| = 0
|−8| = 8
6 Adding Integers
To add integers, think about direction on the number line.
Rules:
• Same signs: add the numbers and keep the sign.
• Different signs: subtract the smaller absolute value from the larger and keep the sign of the number with the larger absolute value.
Examples:
(−3) + (−5) = −8
4 + (−7) = −3
6 + 2 = 8
(−6) + 4 = −2
7 Subtracting Integers
Subtracting an integer is the same as adding its opposite.
a − b = a + (−b)
Examples:
5 − 3 = 5 + (−3) = 2
5 − (−3) = 5 + 3 = 8
−4 − 6 = −4 + (−6) = −10
−4 − (−6) = −4 + 6 = 2
8 Multiplying Integers
Multiply the absolute values and determine the sign.
Rules:
• Same signs → positive
• Different signs → negative
Examples:
(−3) × (−4) = 12
3 × (−4) = −12
(−6) × 5 = −30
7 × 2 = 14
9 Dividing Integers
Divide the absolute values and determine the sign.
Rules:
• Same signs → positive
• Different signs → negative
Examples:
(−12) ÷ (−3) = 4
12 ÷ (−3) = −4
(−20) ÷ 5 = −4
20 ÷ 4 = 5
Integers are everywhere! From temperatures to elevations, from money to scores!
Real-World Examples
• Temperatures: −5°C, 0°C, 10°C
• Elevations: −200 ft (below sea level), 0 ft, 500 ft
• Money: −$20 (debt), $0, $50 (credit)
• Scores: −3, 0, 7
PRACTICE • BUILD CONFIDENCE • GO FURTHER
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What are integers?

Integers are all whole numbers **and their opposites**. The set: `ℤ = {..., −3, −2, −1, 0, 1, 2, 3, ...}`.

- Positive integers: 1, 2, 3, ...
- Zero: 0 (neither positive nor negative)
- Negative integers: −1, −2, −3, ...

Integers do **not** include fractions or decimals.

Number line

Each tick mark is one unit. Numbers to the **right** are greater; numbers to the **left** are smaller. Zero sits in the middle.

Comparing integers

On a number line, the number farther right is greater. Examples: $3 > -2$; $0 > -5$; $-1 < 4$; $-7 < -2$.

Remember: any positive integer is greater than any negative integer, and a “bigger-looking” negative is actually smaller ($-6 < -2$).

Opposites (additive inverses)

Every integer has an **opposite** the same distance from 0 on the other side of the number line. Opposite of 5 is −5. Opposite of −3 is 3. Opposite of 0 is 0. And $a + (-a) = 0$.

Absolute value

Absolute value is the **distance** from 0 on the number line — always non-negative. Notation: $|a|$. Examples: $|5| = 5$, $|-5| = 5$, $|0| = 0$, $|-8| = 8$.

Adding integers

- **Same signs:** add the absolute values, keep the sign. $(-3) + (-5) = -8$. $6 + 2 = 8$.
- **Different signs:** subtract the smaller absolute value from the larger, keep the sign of the number with the larger absolute value. $4 + (-7) = -3$. $(-6) + 4 = -2$.

Subtracting integers

Subtracting an integer is the same as **adding its opposite**: `a − b = a + (−b)`. Examples: 5 − 3 = 5 + (−3) = 2. 5 − (−3) = 5 + 3 = 8. −4 − 6 = −4 + (−6) = −10. −4 − (−6) = −4 + 6 = 2.

Multiplying integers

Multiply the absolute values; sign rule:

- Same signs → positive. Different signs → negative.

Examples: (−3) × (−4) = 12. 3 × (−4) = −12. (−6) × 5 = −30. 7 × 2 = 14.

Dividing integers

Divide the absolute values; sign rule is the same:

- Same signs → positive. Different signs → negative.

Examples: (−12) ÷ (−3) = 4. 12 ÷ (−3) = −4. (−20) ÷ 5 = −4. 20 ÷ 4 = 5.

> **Real-world integers:** temperatures (−5°C, 0°C, 10°C), elevations (−200 ft below sea level, 500 ft above), money (−$20 debt, $50 credit), scores (−3, 0, 7).

10 Practice Questions

Q1. Compute −7 + 12.

Q2. Compute −8 + (−5).

Q3. Compute 4 − 9.

Q4. Compute −6 − (−10).

Q5. Compute (−3) × 8.

Q6. Compute $(-6) \times (-7)$.

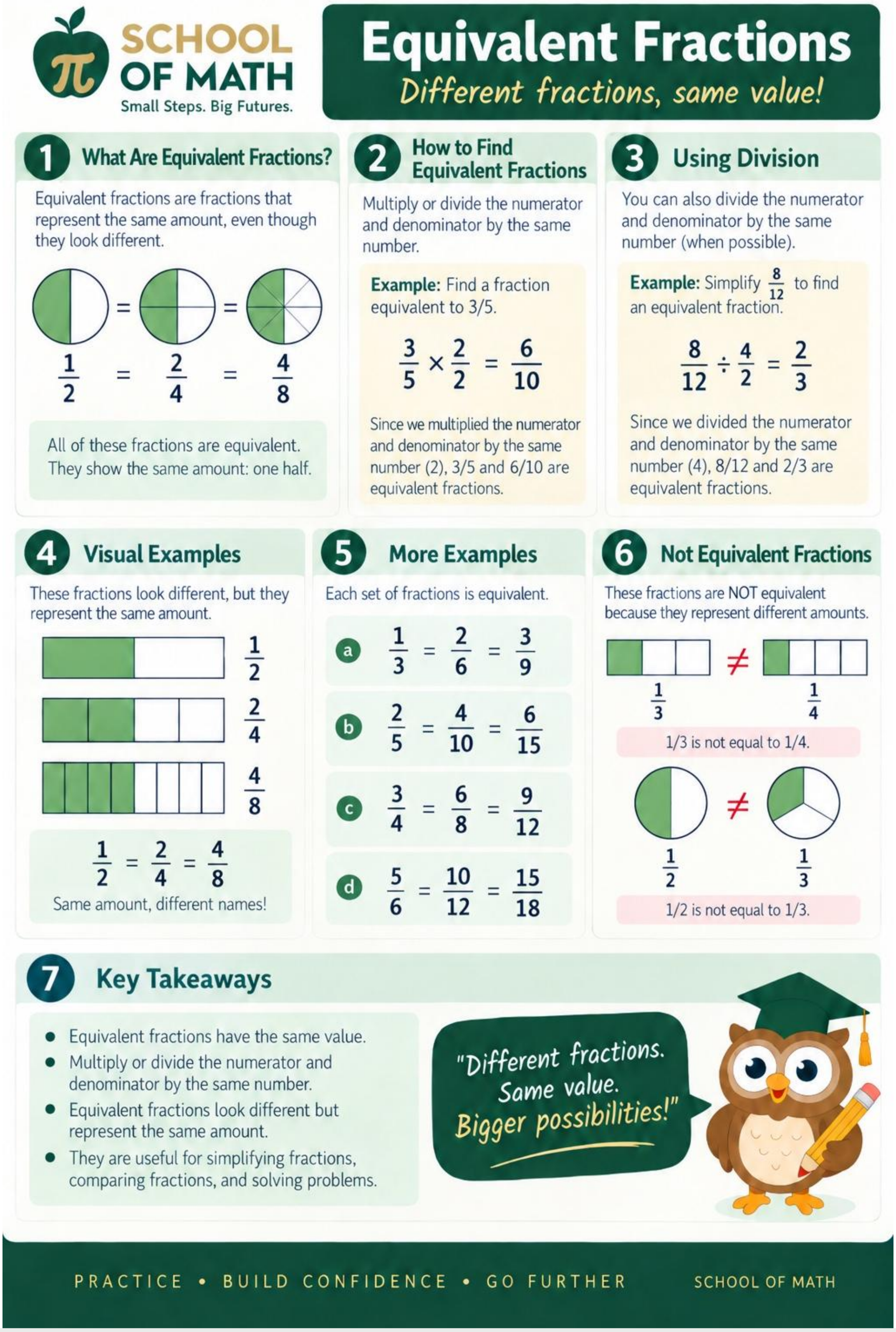
Q7. Compute $(-36) \div 4$.

Q8. Compute $(-56) \div (-7)$.

Q9. Order from least to greatest: $-3, 4, 0, -7, 2$.

Q10. Evaluate $|-15| + |4| - |-9|$.

6. Equivalent Fractions



Equivalent fractions

Equivalent fractions represent the **same amount** even though they look different. Example: 1/2 = 2/4 = 4/8. All three shade the same portion of a whole.

How to find equivalent fractions — multiply

Multiply the numerator and the denominator by the **same** non-zero number.

Example: 3/5 × 2/2 = 6/10. Since we multiplied top and bottom by 2, 3/5 and 6/10 are equivalent.

How to find equivalent fractions — divide (simplify)

Divide the numerator and denominator by the **same** non-zero number (a common factor).

Example: 8/12 ÷ 4/4 = 2/3. Because we divided top and bottom by 4, 8/12 and 2/3 are equivalent.

More equivalent-fraction families

- 1/3 = 2/6 = 3/9
- 2/5 = 4/10 = 6/15
- 3/4 = 6/8 = 9/12
- 5/6 = 10/12 = 15/18

Not equivalent

Fractions with different amounts are not equivalent: 1/3 ≠ 1/4 and 1/2 ≠ 1/3. You can check by cross-multiplication (a/b = c/d ⇔ a×d = b×c).

Why we care

- **Simplifying:** reduce a fraction to lowest terms by dividing by the GCF of numerator and denominator.
- **Comparing fractions:** rewrite with a common denominator, then compare numerators.
- **Adding & subtracting fractions:** requires a common denominator — equivalent fractions get you there.

> **Different fractions. Same value. Bigger possibilities.**

10 Practice Questions

Q1. Write two fractions equivalent to 2/3.

Q2. Simplify 18/24 to lowest terms.

Q3. Fill in: 5/8 = ?/40.

Q4. Are 6/9 and 8/12 equivalent? Show why.

Q5. Are 3/5 and 4/7 equivalent?

Q6. Simplify 45/60.

Q7. Which is larger: 2/3 or 5/8? Rewrite with a common denominator.

Q8. Find a fraction with denominator 100 that is equivalent to 7/20.

Q9. Simplify 24/36 in two steps, using GCF at the end.

Q10. A recipe uses 6/8 cup of flour. Write it in simplest form and give two other equivalent fractions.