

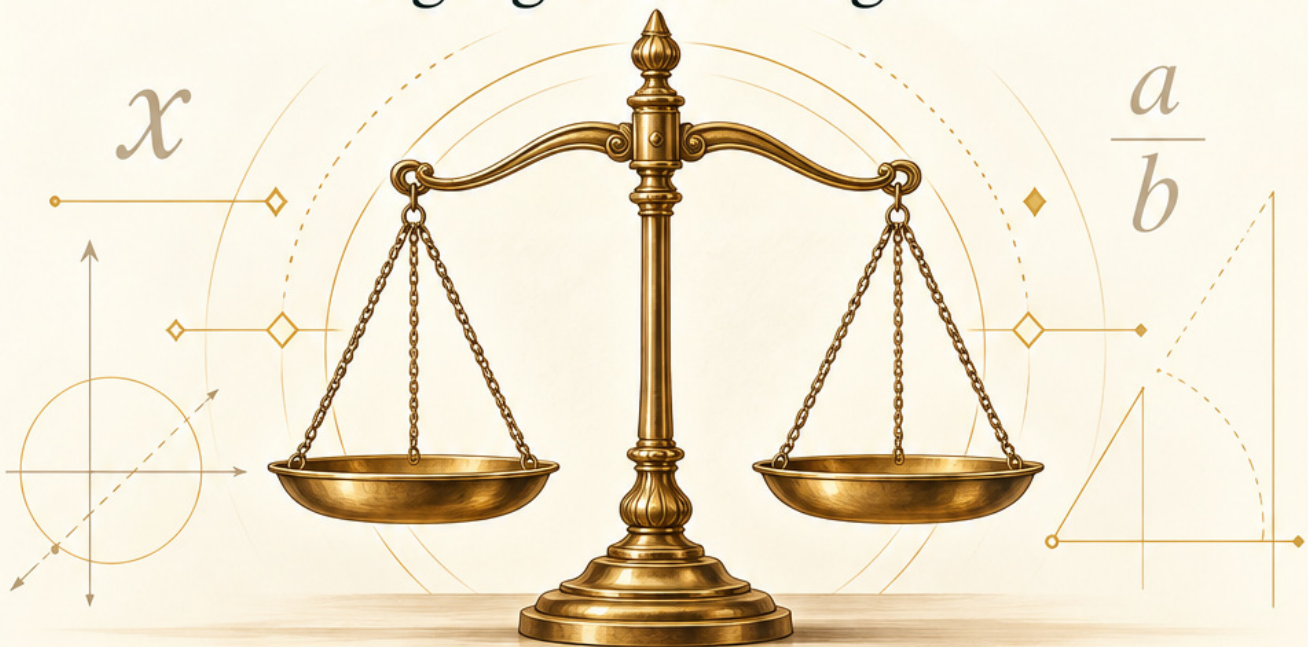


◆ PRE - ALGEBRA ◆

◆ CLASS 15 ◆

Two-Step Equations

◆ Bringing It All Together ◆



$$\begin{aligned} 3x + 5 &= 20 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

Inverse Operations • Fractions & Decimals • Word Problems

1. Understanding Two-Step Equations

Identify the two operations applied to the variable.

$$3x + 5 = 20$$



Two operations: multiplication and addition.

$$\frac{x}{4} - 3 = 6$$

Divide by 4, then subtract 3.



A two-step equation takes **two inverse operations** to isolate the variable.

Image 1 practice: Understanding two-step equations

Q1. Name the two operations applied to x , then solve:

$$2x + 3 = 17$$

Q2. Name the two operations applied to x , then solve:

$$5x - 4 = 26$$

Q3. Name the two operations applied to x , then solve:

$$\frac{x}{3} + 2 = 7$$

Q4. Name the two operations applied to x , then solve:

$$\frac{x}{6} - 1 = 4$$

Q5. Explain which operation to undo first, then solve:

$$4x + 7 = 39$$

2. Using Inverse Operations

Undo addition or subtraction first.
Then undo multiplication or division.

INVERSE OPERATION PAIRS

Addition $\longleftrightarrow$ Subtraction

Multiplication $\longleftrightarrow$ Division

EXAMPLE: SOLVE FOR x

$$3x + 5 = 20$$

1 Subtract 5 from both sides.

$$3x + 5 - 5 = 20 - 5$$

$$3x = 15$$

*We subtract 5 from both sides
to undo the + 5.*

2 Divide both sides by 3.

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

*We divide both sides by 3
to undo the 3.*

$$\text{Check: } 3(5) + 5 = 20 \checkmark$$



KEY TAKEAWAY

Keep the equation balanced:
do the same operation on both sides.

Image 2 practice: Using inverse operations

Q6. State the first inverse operation and solve:

$$3x + 4 = 25$$

Q7. State the first inverse operation and solve:

$$6x - 5 = 31$$

Q8. State both inverse operations in order and solve:

$$\frac{x}{5} + 3 = 11$$

Q9. A student wants to subtract 9 first. Choose the correct first step and solve:

$$7x - 9 = 26$$

Q10. Explain why you add 2 first, then solve:

$$\frac{x}{4} - 2 = 9$$



3. Solving Equations with Multiplication

Undo addition or subtraction, then divide by the coefficient.

Example 1: Addition

$$3x + 5 = 20$$

$$3x = 15$$

$$\frac{3x}{3} = \frac{15}{3}$$

$$x = 5$$

1 Subtract 5 from both sides.

2 Divide both sides by 3.

Example 2: Subtraction

$$4x - 7 = 13$$

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

1 Add 7 to both sides.

2 Divide both sides by 4.

The **coefficient** is the number multiplying the variable.



First remove the constant.
Then divide by the coefficient.



Image 3 practice: Equations with multiplication

Q11. Solve:

$$8x + 5 = 61$$

Q12. Solve:

$$9x - 11 = 43$$

Q13. Solve:

$$6x + 14 = 62$$

Q14. Solve:

$$12x - 7 = 65$$

Q15. Solve:

$$7x + 8 = 43$$

4. Solving Equations with Division

Undo addition or subtraction, then multiply by the divisor.

Example 1: Subtraction



$$\begin{aligned}\frac{x}{4} - 3 &= 6 \\ \frac{x}{4} &= 9 \\ 4 \cdot \frac{x}{4} &= 4 \cdot 9 \\ x &= 36\end{aligned}$$

- 1 Add 3 to both sides.
- 2 Multiply both sides by 4.

Example 2: Addition



$$\begin{aligned}\frac{x}{5} + 2 &= 8 \\ \frac{x}{5} &= 6 \\ 5 \cdot \frac{x}{5} &= 5 \cdot 6 \\ x &= 30\end{aligned}$$

- 1 Subtract 2 from both sides.
- 2 Multiply both sides by 5.

Check: $36 \div 4 - 3 = 6$ ✓



Multiplication undoes division.



Image 4 practice: Equations with division

Q16. Solve:

$$\frac{x}{2} + 6 = 17$$

Q17. Solve:

$$\frac{x}{7} - 4 = 5$$

Q18. Solve:

$$\frac{x}{8} + 3 = 12$$

Q19. Solve:

$$\frac{x}{9} - 5 = 3$$

Q20. Solve:

$$\frac{x}{6} + 7 = 15$$



5. Working with Negative Numbers

Use inverse operations and track each negative sign.

Example 1: Negative Coefficient



$$-3x + 5 = 20$$

$$-3x = 15$$

$$\frac{-3x}{-3} = \frac{15}{-3}$$

$$x = -5$$

1 Subtract 5 from both sides.

2 Divide both sides by -3 .

Example 2: Negative Constant



$$4x - 7 = -19$$

$$4x = -12$$

$$\frac{4x}{4} = \frac{-12}{4}$$

$$x = -3$$

1 Add 7 to both sides.

2 Divide both sides by 4.

Check: $-3(-5) + 5 = 20$ ✓

◆ Same signs: positive quotient. Different signs: negative quotient. ◆

Image 5 practice: Working with negative numbers

Q21. Solve:

$$-4x + 6 = 26$$

Q22. Solve:

$$5x - 9 = -29$$

Q23. Solve:

$$-6x - 7 = 23$$

Q24. Solve:

$$\frac{x}{-3} + 4 = 10$$

Q25. Solve:

$$-2x + 9 = -7$$



6. Working with Fractions and Decimals

Use the same inverse operations with every number type.

Example 1: Fractions



$$\frac{1}{2}x + \frac{3}{4} = \frac{11}{4}$$

$$\frac{1}{2}x = 2$$

$$2 \cdot \frac{1}{2}x = 2 \cdot 2$$

$$x = 4$$

1 Subtract $\frac{3}{4}$ from both sides.

2 Multiply both sides by 2.

Example 2: Decimals



$$0.5x + 1.2 = 3.7$$

$$0.5x = 2.5$$

$$\frac{0.5x}{0.5} = \frac{2.5}{0.5}$$

$$x = 5$$

1 Subtract 1.2 from both sides.

2 Divide both sides by 0.5.

Check: $0.5(5) + 1.2 = 3.7$ ✓

◆ Fractions and decimals follow the same solving steps. ◆

Image 6 practice: Working with fractions and decimals**Q26. Solve:**

$$\frac{1}{3}x + \frac{1}{2} = \frac{5}{2}$$

Q27. Solve:

$$\frac{3}{4}x - \frac{1}{2} = 4$$

Q28. Solve:

$$0.4x + 1.6 = 4.4$$

Q29. Solve:

$$1.5x - 2.4 = 6.6$$

Q30. Solve:

$$\frac{x}{2.5} + 0.8 = 4.8$$

7. Variable on the Right

The same solving steps work on either side of the equals sign.

Example 1: Multiplication



$$17 = 4x + 1$$

$$16 = 4x$$

$$\frac{16}{4} = \frac{4x}{4}$$

$$4 = x$$

1 Subtract 1 from both sides.

2 Divide both sides by 4.

So, $x = 4$.

Example 2: Division



$$6 = \frac{x}{3} - 2$$

$$8 = \frac{x}{3}$$

$$3 \cdot 8 = 3 \cdot \left(\frac{x}{3}\right)$$

$$24 = x$$

1 Add 2 to both sides.

2 Multiply both sides by 3.

So, $x = 24$.

Check: $17 = 4(4) + 1$ ✓

◆ If $4 = x$, then $x = 4$. Equality works both ways. ◆

Image 7 practice: Variable on the right**Q31. Solve:**

$$37 = 5x + 2$$

Q32. Solve:

$$16 = 6x - 8$$

Q33. Solve:

$$12 = \frac{x}{4} + 5$$

Q34. Solve:

$$19 = -3x + 4$$

Q35. Solve:

$$4.9 = 0.5x + 1.4$$

8. Solving Word Problems

Define the variable, write an equation, and solve.

A Visit to the Museum

A museum charges an \$8 entry fee plus \$4 for each special exhibit. You spend \$24 in total. How many special exhibits did you visit?



1

Define the variable

Let x be the number of special exhibits.

2

Write the equation

$$4x + 8 = 24$$

Exhibit cost + entry fee = total cost

3

Solve

$$4x = 16$$

$$x = 4$$

Subtract 8 from both sides.

Divide both sides by 4.

You visited 4 special exhibits.

Check: $4(4) + 8 = 24$ ✓



State your answer in the context of the problem.



Image 8 practice: Solving word problems

Q36. An activity center charges a \$6 entry fee plus \$3 per game. You spend \$27 in total. How many games do you play? Define x , write a two-step equation, and solve.

Q37. An art workshop charges \$4 for supplies and \$6 per lesson. The total bill is \$34. How many lessons are included? Define x , write a two-step equation, and solve.

Q38. Four identical notebooks cost \$25 after a \$3 discount on the entire purchase. What was the original price of one notebook? Define x , write a two-step equation, and solve.

Q39. Some tokens are shared equally among 5 players. One player then receives 2 extra tokens and has 9 tokens. How many tokens were shared originally? Define x , write a two-step equation, and solve.

Q40. A reader has already read 15 pages and then reads 12 pages each day. The reader reaches 75 pages read. How many additional days of reading were needed? Define x , write a two-step equation, and solve.

Homework review: R1–R5

R1. Identify the two operations applied to x , then solve:

$$7x + 3 = 52$$

R2. State the inverse operations in order and solve:

$$\frac{x}{8} - 3 = 4$$

R3. Solve:

$$9x + 8 = 71$$

R4. Solve:

$$\frac{x}{6} + 4 = 13$$

R5. Solve:

$$-5x - 2 = 18$$

Homework review: R6–R8

R6. Solve:

$$\frac{1}{4}x + \frac{1}{2} = \frac{7}{2}$$

R7. Solve:

$$23 = 3x - 4$$

R8. A bird park charges a \$10 group booking fee and \$3 per student. The total is \$31. How many students are in the group? Define x , write a two-step equation, and solve.

Homework word problems: W1–W5

W1. Mia starts with \$7 and saves \$3 each week. She now has \$22. For how many weeks has she saved? Define x , write a two-step equation, and solve.

W2. A fair charges a \$5 entrance fee and \$4 per ride. A visitor spends \$29. How many rides did the visitor take? Define x , write a two-step equation, and solve.

W3. Six identical notebooks cost \$32 after a \$4 coupon is applied to the whole purchase. What is the price of one notebook before the coupon? Define x , write a two-step equation, and solve.

W4. A set of cards is shared equally among 4 children. One child receives 3 bonus cards and then has 11. How many cards were in the original set? Define x , write a two-step equation, and solve.

W5. Jon has read 14 pages of a book. He reads 8 more pages each day until he has read 70 pages. How many additional days does he read? Define x , write a two-step equation, and solve.

Homework word problems: W6–W10

W6. Five students pay the same admission price for a museum trip. Their group also pays a \$12 booking fee. The total bill is \$57. What is the admission price per student? Define x , write a two-step equation, and solve.

W7. A player starts with 12 points and loses 3 points in each round. After several rounds, the score is -6 . How many rounds were played? Define x , write a two-step equation, and solve.

W8. A freezer starts at 4°C and cools at a constant rate of 3°C per minute. How many minutes does it take to reach -14°C ? Define x , write a two-step equation, and solve.

W9. A fictional shuttle fare is a \$4.50 starting fee plus \$2.50 per mile. The total fare is \$19.50. How many miles is the trip? Define x , write a two-step equation, and solve.

W10. A craft project uses half of a ribbon plus 1.25 meters from a second roll. The project uses 4.75 meters altogether. How long was the first ribbon? Define x , write a two-step equation, and solve.

Homework word problems: W11–W12

W11. A tank contains two-thirds of its full capacity. After 4 liters are added, it contains 18 liters. What is the tank's full capacity? Define x , write a two-step equation, and solve.

W12. A rope is cut into 6 equal pieces. One piece is shortened by 1.5 meters and is then 5.5 meters long. How long was the original rope? Define x , write a two-step equation, and solve.