



SCHOOL
OF MATH

PRE-ALGEBRA

CLASS 16

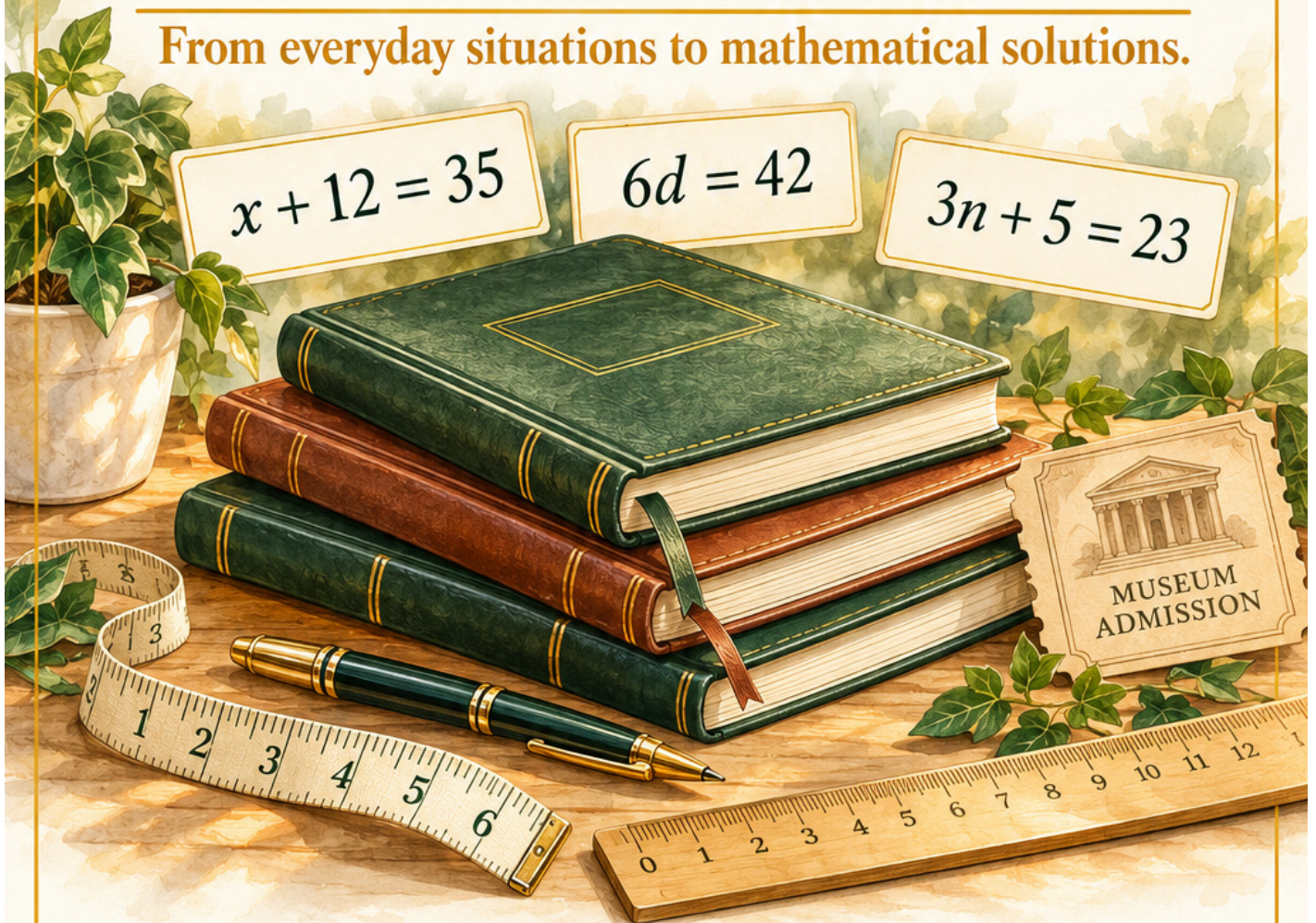
Word Problems Modeled with One- and Two-Step Equations

From everyday situations to mathematical solutions.

$$x + 12 = 35$$

$$6d = 42$$

$$3n + 5 = 23$$



Define • Model • Solve • Check

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1. Identifying the Unknown

Choose a variable. Explain what it represents.



The **unknown** is the quantity a problem asks you to find.
A **variable** is a letter that represents that quantity.

1

Read the question carefully.

2

Identify the missing quantity.

3

Define a variable with units.

WORKED EXAMPLE

A library display has some books. After 8 more books are added, there are 23 books. How many books were there at the start?



KNOWN INFORMATION

- 8 books were added.
- 23 books are on display now.

UNKNOWN QUANTITY

The number of books at the start.

DEFINE A VARIABLE

Let b = the number of books at the start.

EQUATION FOR THE SITUATION

$$b + 8 = 23$$

The variable represents the starting amount.

WHAT DOES THE VARIABLE REPRESENT?

1

A ribbon is cut into 4 equal pieces. How long is each piece?



Let L = the length of each piece, in centimeters.

2

You buy several \$3 notebooks. How many notebooks did you buy?



Let n = the number of notebooks.

KEY TAKEAWAY:

Define the quantity, not just the object: n = number of notebooks.

Image 1 practice: Identifying the unknown

Q1. A jar has some marbles. After 9 are added, it has 31. Define m for the question “How many marbles were in the jar at first?” Do not solve yet.

Q2. Five identical folders cost \$20. Define a variable for the price of one folder, and include its units.

Q3. A rope is cut into 6 equal pieces. Each piece is 7 cm long. Define L for the original rope length.

Q4. A box contains 52 cards. Some are given away, leaving 37. Define c for the amount given away and list the two known counts.

Q5. A club pays a \$12 booking fee plus \$4 per player. Its bill is \$48. A student writes “ p is the price.” Replace this with a precise variable definition for finding how many players attended.

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2. Translating Words into Equations

Turn a sentence into a mathematical statement.



An **equation** states that two expressions are equal.
Use a **variable** for the unknown quantity.

WORDS → EQUATION

Verbal Statement	Equation
A number plus 6 is 19.	$x + 6 = 19$
A number decreased by 4 is 11.	$x - 4 = 11$
Three times a number is 24.	$3x = 24$
A number divided by 5 is 7.	$x \div 5 = 7$

WORKED EXAMPLE

Maya buys 3 sketchbooks and a \$4 pencil set. She spends \$19 in total. Each sketchbook costs the same amount. What is the price of one sketchbook?



1 DEFINE

Let s = the price of one sketchbook, in dollars.

2 TRANSLATE

3 sketchbooks $\rightarrow 3s$
plus \$4 $\rightarrow + 4$
total \$19 $\rightarrow = 19$

3 WRITE THE EQUATION

$$3s + 4 = 19$$

ORDER MATTERS



The order of the words tells you which quantity comes first in the equation.

5 less than a number

$$x - 5$$

5 minus a number

$$5 - x$$

YOUR TURN

- 1 Twice a number plus 7 equals 21.
Write an equation using n .

Image 2 practice: Translating words into equations

Q6. Write an equation using n : a number increased by 11 equals 29. Do not solve.

Q7. Write an equation using n : 7 less than a number is 18. Do not solve.

Q8. Write an equation using n : 26 minus a number is 9. Do not solve.

Q9. A shop charges \$6 for delivery plus \$4 for each box. The total is \$34. Define b and write an equation; do not solve.

Q10. Write an equation for each statement: (a) twice the sum of n and 5 equals 24; (b) five more than twice n equals 24. Do not solve.

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3. Addition Word Problems

Find an unknown starting amount or missing part.



When an amount **increases**, use **addition** to model the change.

MODEL

$$\text{starting amount} + \text{amount added} = \text{total}$$

WORKED EXAMPLE

A school art club has some paintbrushes. It receives 12 more paintbrushes and now has 35. How many paintbrushes did it have at the start?



1 DEFINE THE UNKNOWN

Let p = the number of paintbrushes at the start.

2 WRITE AN EQUATION

$$p + 12 = 35$$

3 USE THE INVERSE OPERATION

$$\begin{aligned} p + 12 - 12 &= 35 - 12 \\ p &= 23 \end{aligned}$$

4 CHECK AND ANSWER

$23 + 12 = 35$ ✓
The club had 23 paintbrushes at the start.

BAR MODEL

Unknown part + known part = whole

35 paintbrushes



KEY TAKEAWAY

Addition builds the equation.
Subtraction finds the missing addend.

YOUR TURN

- 1** You save \$18 more and now have \$50.
How much money did you have before?
Let m = the amount of money you had before, in dollars.

Equation: _____

m = _____

Image 3 practice: Addition: finding the missing part

Q11. A shelf gains 7 books and now holds 26. Write and solve an equation for the number it held before.

Q12. Lena adds \$16 to her savings and then has \$61. Find her starting savings using an equation.

Q13. A team has 28 points and finishes with 47 after one round. How many points did it earn in that round? Write and solve an equation.

Q14. After 2.75 liters of water are added, a container holds 8.5 liters. Find the original amount using an equation.

Q15. An afternoon temperature is 9°C higher than the morning temperature. The afternoon temperature is 4°C . Find the morning temperature.

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4. Subtraction Word Problems

Find an original amount or how much was removed.



Use **subtraction** to model an amount being taken away.

MODEL

$$\text{starting amount} - \text{amount removed} = \text{amount left}$$

WORKED EXAMPLE

A museum shop has some postcards. It sells 18 postcards and has 27 left. How many postcards did it have before the sale?



1 DEFINE THE UNKNOWN

Let p = the number of postcards before the sale.

2 WRITE AND SOLVE

$$\begin{aligned} p - 18 &= 27 \\ p - 18 + 18 &= 27 + 18 \\ p &= 45 \end{aligned}$$

3 CHECK AND ANSWER

$$45 - 18 = 27 \checkmark$$

The shop had 45 postcards before the sale.

ANOTHER EXAMPLE

A ribbon is 50 cm long. After a piece is cut off, 32 cm remains. How much ribbon was cut off?

When the amount removed is unknown



1 DEFINE THE UNKNOWN

Let c = the length cut off, in centimeters.

2 WRITE AND SOLVE

$$\begin{aligned} 50 - c &= 32 \\ c &= 18 \end{aligned}$$

3 CHECK AND ANSWER

$$50 - 18 = 32 \checkmark$$

18 cm was cut off.



KEY TAKEAWAY

Locate the **unknown** before choosing how to solve.

Unknown start:

$$x - 8 = 15 \rightarrow x = 23$$

Unknown amount removed:

$$23 - x = 15 \rightarrow x = 8$$

YOUR TURN

1 You spend \$14 and have \$29 left. How much money did you have at first? Let m = your starting amount, in dollars.

Equation: _____

m = _____

Image 4 practice: Subtraction: start or amount removed

Q16. A basket loses 11 apples and has 24 left. How many apples were in it at first? Write and solve an equation.

Q17. A class begins with 63 stickers and gives some away, leaving 38. Find the number given away.

Q18. After a \$27 purchase, Omar has \$46 left. Find how much money he had before the purchase.

Q19. A 12.5-meter cable is shortened to 8.75 meters. Write and solve an equation for the length removed.

Q20. A freezer temperature falls by 8°C and becomes -3°C . What was its temperature before the change?

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5. Multiplication Word Problems

Model equal groups and repeated quantities.



Multiplication connects the number of equal groups, the amount in each group, and the total.

MODEL

$$\text{number of groups} \times \text{amount in each group} = \text{total}$$

WORKED EXAMPLE

A gallery places 6 photographs in each display. There are 42 photographs altogether. How many displays are there?



1 DEFINE THE UNKNOWN

Let d = the number of displays.

2 WRITE AND SOLVE

$$\begin{aligned} 6d &= 42 \\ 6d \div 6 &= 42 \div 6 \\ d &= 7 \end{aligned}$$

3 CHECK AND ANSWER

$$6 \times 7 = 42 \checkmark$$

There are 7 displays.

ANOTHER EXAMPLE

When the amount in each group is unknown

Four identical notebooks cost \$28 in total. What is the price of one notebook?



1 DEFINE THE UNKNOWN

Let n = the price of one notebook, in dollars.

2 WRITE AND SOLVE

$$4n = 28 \rightarrow n = 7$$

3 CHECK AND ANSWER

$$4 \times 7 = 28 \checkmark$$

Each notebook costs \$7.



KEY TAKEAWAY

Multiplication models equal groups.
Division finds an unknown factor.

NOTATION

$6d$ means $6 \times d$.

YOUR TURN

- 1 A theater has 8 seats in each row and 96 seats altogether. How many rows are there?
Let r = the number of rows.

Equation: _____
 $r =$ _____

Image 5 practice: Multiplication: equal groups

Q21. Five bags each contain the same number of beads. There are 45 beads altogether. Find the number in each bag.

Q22. A tray holds 7 cupcakes. A baker fills identical trays with 56 cupcakes in total. How many trays are filled?

Q23. Six identical sketchpads cost \$51 altogether. Find the price of one sketchpad.

Q24. A fabric strip is 3 times as long as a 4.2-meter strip. Define L as the longer length and write and solve an equation.

Q25. A rectangle has area 84 square centimeters and width 7 centimeters. Find its length using an equation.

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6. Division Word Problems

Model equal sharing and find the number of groups.



Division describes how a total is split into equal groups.

MODELS

total $\div$ number of groups =
amount in each group

total $\div$ amount in each group =
number of groups

WORKED EXAMPLE

Equal sharing: find the original total

A pottery studio divides its clay equally among 6 students. Each student receives 4 pounds. How many pounds of clay did the studio divide?



1 DEFINE THE UNKNOWN

Let c = the total weight of clay, in pounds.

2 WRITE AND SOLVE

$$\begin{aligned} c \div 6 &= 4 \\ (c \div 6) \times 6 &= 4 \times 6 \\ c &= 24 \end{aligned}$$

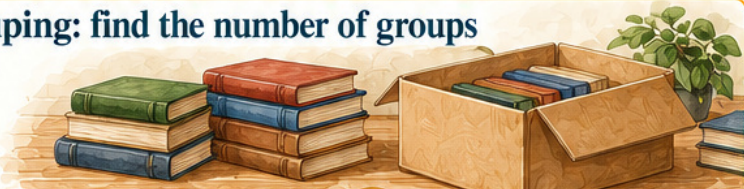
3 CHECK AND ANSWER

$24 \div 6 = 4$ ✓
The studio divided 24 pounds of clay.

ANOTHER EXAMPLE

Grouping: find the number of groups

A librarian packs 36 books into boxes, with 9 books in each box. How many boxes are needed?



1 DEFINE THE UNKNOWN

Let b = the number of boxes.

2 WRITE AND SOLVE

$$\begin{aligned} 36 \div 9 &= b \\ b &= 4 \end{aligned}$$

3 CHECK AND ANSWER

$4 \times 9 = 36$ ✓
4 boxes are needed.



KEY TAKEAWAY

Identify the total, the number of groups, and the amount in each group.

NOTATION

Multiplication undoes division.

YOUR TURN

1 A ribbon is cut into 5 equal pieces. Each piece is 8 cm long. What was the total length of the ribbon?
Let L = the total ribbon length, in centimeters.

Equation: _____

L = _____

Image 6 practice: Division: equal sharing and grouping

Q26. A ribbon is divided into 7 equal pieces, each 6 cm long. Find the original length.

Q27. A teacher shares 72 counters equally among 8 students. Find each student's share using a division equation.

Q28. A gardener places 54 seedlings into pots, with 6 seedlings per pot. How many pots are used?

Q29. A 45-liter supply is shared equally among some tanks, giving each tank 5 liters. Find the number of tanks.

Q30. A bag of rice is shared equally among 4 families. Each receives 1.75 kg. Find the original mass of rice.

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7. Two-Step Word Problems

Combine two operations in one equation.



A **two-step equation** needs two inverse operations to isolate the variable.

WORKED EXAMPLE

Shopping: a fixed cost plus equal items

You buy 3 identical notebooks and a \$5 pen. The total is \$23. What is the price of one notebook?



1 DEFINE

Let n = the price of one notebook, in dollars.

2 MODEL

$$3n + 5 = 23$$

3 SOLVE

$$\begin{aligned} 3n + 5 - 5 &= 23 - 5 \\ 3n &= 18 \\ n &= 6 \end{aligned}$$

4 CHECK

$$3(6) + 5 = 23 \checkmark$$

Each notebook costs \$6.

WORKED EXAMPLE

Measurement: repeated lengths minus a piece

A ribbon is 4 times the length of a bookmark. After 6 cm is cut off the ribbon, 30 cm remains. How long is the bookmark?



1 DEFINE

Let b = the bookmark length, in centimeters.

2 MODEL

$$4b - 6 = 30$$

3 SOLVE

$$\begin{aligned} 4b &= 36 \\ b &= 9 \end{aligned}$$

4 CHECK

$$4(9) - 6 = 30 \checkmark$$

The bookmark is 9 cm long.



KEY TAKEAWAY

Undo operations in reverse order.

First undo addition or subtraction.
Then undo multiplication or division.

YOUR TURN

- 1 A taxi charges a \$4 starting fee plus \$3 per mile. The fare is \$25. How many miles was the ride?
Let m = the number of miles.

Equation: _____

m = _____

Image 7 practice: Modeling two-step situations

Q31. Four identical snack boxes and a \$7 delivery fee cost \$39. Find the price of one box.

Q32. Five equal pieces are joined end to end. After 9 cm is trimmed off, the result is 46 cm long. Find the length of each original piece.

Q33. A bag of counters is shared equally among 6 students. Each then receives 3 extra counters and has 11. Find the number in the original bag.

Q34. A roll of fabric has 8 meters removed. The rest is cut into 4 equal pieces of 7 meters each. Find the original length.

Q35. A bicycle rental costs \$4.50 to start plus \$2.75 per hour. The charge is \$21, with no other fees. Find the rental time.

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8. Checking and Interpreting Solutions

Verify your answer. Explain what it means.



A **solution** must make the equation true and **make sense** in the original problem.

1 Substitute the value.

Replace the variable with the proposed value in the equation.

2 Check both sides.

Simplify each side to see if the equation is true.

3 Answer with units.

Explain what the solution means in the context of the problem.

WORKED EXAMPLE

A museum: admission plus activities

A museum charges \$4 admission plus \$3 per activity. You spend \$19. How many activities did you choose?



1 DEFINE

Let a = the number of activities.

2 MODEL

$$3a + 4 = 19$$

3 SOLVE

$$\begin{aligned} 3a + 4 &= 19 \\ 3a &= 15 \\ a &= 5 \end{aligned}$$

4 CHECKING THE SOLUTION

1. Substitute

$$3(5) + 4 = 19$$

2. Calculate

$$\begin{aligned} 15 + 4 &= 19 \\ 19 &= 19 \checkmark \end{aligned}$$

3. Interpret

You chose 5 activities.

SPOT THE ERROR

Four notebooks and a pencil

Four identical notebooks and a \$2 pencil cost \$22 altogether.



1 DEFINE

Let n = the price of one notebook, in dollars.

2 MODEL

$$4n + 2 = 22$$

3 CHECK TWO VALUES

Test $n = 6$

$$4(6) + 2 = 26$$

$$26 \neq 22 \text{ ✗}$$

6 is **not** a solution.

Test $n = 5$

$$4(5) + 2 = 22$$

$$22 = 22 \checkmark$$

Each notebook costs \$5.

★ DOES IT MAKE SENSE?

- ☐ Does the answer match the question?
- ☐ Are the units correct?
- ☐ Is the quantity reasonable?

YOUR TURN

- 1 A club has 3 equal teams and 2 extra players, for a total of 26 players. Let p = the number of players on each team. $3p + 2 = 26$. Is $p = 8$ correct? Check and write a sentence.

Image 8 practice: Checking and interpreting solutions

Q36. A ticket booth charges \$5 plus \$2 per game. A player spends \$19. Check the claim that the player chose 7 games, and answer in a sentence.

Q37. Three identical plants and a \$4 pot cost \$31. A student says each plant costs \$11. Check and correct the answer.

Q38. A 20-meter cord is cut into 8 equal pieces. A student solves $8L = 20$ and writes “ $L = 2.5$ pieces.” Correct the interpretation.

Q39. An organizer says 3 equal teams plus 2 extra players total 24 players. Solve $3p + 2 = 24$ and explain whether a whole-number team size fits.

Q40. A service charges a \$9 setup fee plus \$5 per hour, totaling \$34. A student solves $9h + 5 = 34$. Explain the modeling error and find the correct time.

Homework review: R1–R5

R1. A store starts with 80 postcards and has 53 after selling some. Define s for the unknown amount sold. Do not solve.

R2. Write an equation using n : 4 less than three times a number is 32. Do not solve.

R3. A reading log gains 18 minutes and now shows 65 minutes. Find the previous total.

R4. A box holds 92 buttons. Some are removed, leaving 57. Find how many were removed.

R5. Seven equal packs contain 91 cards altogether. Find the number in one pack.

Homework review: R6–R8

R6. A batch of juice is shared among 5 jugs, with 2.4 liters in each. Find the original volume.

R7. Four identical craft kits and a \$9 fee cost \$57. Find the price of one kit.

R8. Six equal ribbon pieces plus a 5 cm offcut total 50 cm. A student says each piece is 8 cm long. Check and correct the answer.

Homework word problems: W1–W5

W1. A birdwatcher records some birds, then spots 13 more. The final count is 42. Define a variable and find the first count.

W2. An aquarium uses 17 liters of water from a supply and has 36 liters left. Find the initial supply.

W3. A volunteer packs 84 meals equally into 7 crates. Find the number of meals per crate.

W4. A batch of paint fills 9 identical jars with 0.6 liters each. Find the original paint volume.

W5. A school buys 5 identical balls and pays a \$6 delivery charge. Its total is \$71. Find the price of one ball.

Homework word problems: W6–W10

W6. Eight identical notebooks cost \$46 after a \$10 discount on the entire purchase. Find the original price of one notebook.

W7. A bag of shells is shared among 4 children. Each then collects 5 more and has 17. Find the original number of shells in the bag.

W8. A teacher sets aside 9 markers, then divides the rest equally among 7 tables. Each table receives 6 markers. Find the original marker count.

W9. A taxi costs \$6.25 to start and \$2.50 per mile. The total is \$25, with no other charges. Find the distance.

W10. A rectangle has width 6 cm and perimeter 34 cm. Write and solve a two-step equation for its length.

Homework word problems: W11–W12

W11. A family buys 4 child tickets and one \$18 adult ticket. It uses a \$6 coupon on the whole purchase and pays \$52. Combine the fixed amounts, then find the price of one child ticket.

W12. A 30 cm strip is to make 7 equal bookmarks with exactly 3 cm left over. A planner claims each bookmark can have a whole-number length in centimeters. Test the claim; if false, give the exact length required.