

CLASS 17

Solving Multi-Step Equations

Variables on Both Sides



$$5x + 7 = 2x + 19$$

SIMPLIFY • BALANCE • SOLVE • CHECK

Build confidence, one step at a time.

1. Understanding Balance & Equivalent Equations

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An **equation** states that two expressions have the same value.



Both sides are equal.

SOLVING THE EQUATION

$$3x + 6 = x + 14 \longrightarrow \text{Subtract } x \text{ from BOTH sides.}$$

$$2x + 6 = 14 \longrightarrow \text{Subtract 6 from BOTH sides.}$$

$$2x = 8 \longrightarrow \text{Divide BOTH sides by 2.}$$

$$x = 4$$



Keep the balance: perform the same operation on both sides.

Check: $3(4) + 6 = 4 + 14 \rightarrow 18 = 18$

Understanding balance and equivalent equations

Q1. Solve for x . Show the operation applied to both sides.

$$2x + 7 = x + 12$$

Q2. Solve and explain why subtracting $2x$ from both sides preserves equality.

$$4x + 3 = 2x + 17$$

Q3. Solve for x . Show the operation applied to both sides.

$$5x - 6 = 3x + 10$$

Q4. A student subtracts $2x$ only from the left side. Explain the error, then solve correctly.

$$6x + 5 = 2x + 29$$

Q5. Solve. Explain why it is valid to collect the variables on either side.

$$2x + 15 = 5x - 6$$

2. Combining Like Terms on Each Side

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Simplify each side before collecting variables.

Like terms have the same variable raised to the same power.

Examples:

$$3x + x = 4x$$

$$7 - 2 = 5$$

Keep variable terms and constants separate.

SOLVING THE EQUATION

$$3x + 2 + x = 2x + 10 \rightarrow$$

Combine like terms on the **left side**.

Combine like terms on the same side.

$$4x + 2 = 2x + 10 \rightarrow$$

Simplify the **left side**.

$$2x + 2 = 10 \rightarrow$$

Subtract $2x$ from **BOTH** sides.

$$2x = 8 \rightarrow$$

Subtract 2 from **BOTH** sides.

$$x = 4 \rightarrow$$

Divide **BOTH** sides by 2.

ANOTHER EXAMPLE: SIMPLIFY **BOTH** SIDES

$$5x + 7 - 2 = x + 3 + x \rightarrow$$

$$5x + 5 = 2x + 3$$



Combine like terms within each side.
Use the same operation on both sides to keep equality.

Check: $3(4) + 2 + 4 = 2(4) + 10 \rightarrow 18 = 18$

Combining like terms on each side

Q6. Solve for x . Show the operation applied to both sides.

$$2x + 3x + 4 = x + 24$$

Q7. Solve for x . Show the operation applied to both sides.

$$7x - 2x + 8 = 2x + 29$$

Q8. Solve for x . Show the operation applied to both sides.

$$4x + 9 - 3 = 2x + x + 18$$

Q9. Solve for x . Show the operation applied to both sides.

$$8x - 3x + 10 - 4 = 2x + x + 22$$

Q10. Simplify both sides, solve, and check the original unsimplified equation.

$$6x - 2x - 9 + 5 = 3x + x - 2x + 14$$

3. Collecting Variables on One Side

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Use **addition** or **subtraction** to gather variable terms on one side.

Subtract the same variable term from **BOTH** sides to keep the equation balanced.

Examples:

$$3x + 5 = 2x + 5$$

$$4x - x = 3x$$

SOLVING THE EQUATION

$$5x + 7 = 2x + 19$$

$$5x + 7 - 2x = 2x + 19 - 2x$$

$$3x + 7 = 19$$

$$3x + 7 - 7 = 19 - 7$$

$$3x = 12$$

$$x = 4$$

Start with the equation.

Subtract **2x** from both sides.

Combine like terms.

Subtract **7** from both sides.

Simplify.

Divide both sides by **3**.

VARIABLES CAN BE ON THE RIGHT, TOO

$$4x + 9 = 6x + 1$$

$$9 = 2x + 1$$

$$8 = 2x$$

$$x = 4$$

Start with the equation.

Subtract **4x** from both sides.

Subtract **1** from both sides.

Divide both sides by **2**.



Choose the side that makes the next steps easier.

Check: $5(4) + 7 = 2(4) + 19 \rightarrow 27 = 27$

Collecting variables on one side

Q11. Solve for x . Show the operation applied to both sides.

$$4x + 8 = x + 23$$

Q12. Solve for x . Show the operation applied to both sides.

$$9x - 4 = 5x + 24$$

Q13. Solve for x . Show the operation applied to both sides.

$$3x + 25 = 7x + 1$$

Q14. Solve for x . Show the operation applied to both sides.

$$2x - 9 = 5x + 6$$

Q15. Solve and leave the answer as an exact fraction.

$$8x + 11 = 2x - 4$$

4. Solving Equations with Parentheses

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Distribute first, **simplify**, then **solve**.

THE DISTRIBUTIVE PROPERTY

$$a(b + c) = ab + ac$$

$$3(x + 2) = 3x + 6$$

Multiply **every term** inside the parentheses.

SOLVING THE EQUATION

$$3(x + 2) = 2(x + 5)$$

$$3x + 6 = 2x + 10$$

$$x + 6 = 10$$

$$x = 4$$

Start with the equation.

Distribute on **BOTH** sides.

Subtract **2x** from both sides.

Subtract **6** from both sides.

ANOTHER EXAMPLE

$$2(x + 4) + 3 = x + 15$$

$$2x + 11 = x + 15$$

$$x = 4$$

Start with the equation.

Distribute and combine constants.

Subtract **x**, then subtract **11**.



Distribute to **every term** before combining like terms.

Check: $3(4 + 2) = 2(4 + 5) \rightarrow 18 = 18$

Solving equations with parentheses

Q16. Solve for x . Show the operation applied to both sides.

$$2(x + 5) = x + 17$$

Q17. Solve for x . Show the operation applied to both sides.

$$4(x - 1) = 2(x + 6)$$

Q18. Solve for x . Show the operation applied to both sides.

$$3(x + 4) + 2 = 2(x + 9)$$

Q19. Solve for x . Show the operation applied to both sides.

$$5(2x - 3) = 3(x + 2)$$

Q20. Solve for x . Show the operation applied to both sides.

$$2(3x - 4) + 5 = 4(x + 2) - 1$$

5. Working with Negative Signs

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Distribute negative coefficients **carefully**.

MULTIPLY EVERY TERM

$$-2(x - 3) = -2x + 6$$

$$-2 \times x = -2x \quad | \quad -2 \times (-3) = +6$$

SOLVING THE EQUATION

$$4 - 2(x - 3) = 3x + 5$$

$$4 - 2x + 6 = 3x + 5$$

$$10 - 2x = 3x + 5$$

$$10 = 5x + 5$$

$$5 = 5x$$

$$x = 1$$

Start with the equation.

Distribute **-2** to **BOTH** terms.

Combine constants on the left.

Add **2x** to both sides.

Subtract **5** from both sides.

Divide both sides by **5**.

AVOID THIS COMMON MISTAKE



Incorrect: ~~$-(x - 4) = -x - 4$~~



Correct: $-(x - 4) = -x + 4$

A minus sign before parentheses means **multiply every term by -1**.



Check: $4 - 2(1 - 3) = 3(1) + 5 \rightarrow 8 = 8$

Working with negative signs

Q21. Solve for x . Show the operation applied to both sides.

$$9 - (x + 2) = 2x + 1$$

Q22. Solve for x . Show the operation applied to both sides.

$$7 - 2(x - 4) = 3x$$

Q23. Solve for x . Show the operation applied to both sides.

$$-3(x + 2) + 8 = x - 10$$

Q24. Solve for x . Show the operation applied to both sides.

$$4 - 2(3x - 5) = x + 7$$

Q25. Distribute all negative factors carefully and solve.

$$5 - 3(2x - 1) = 2(x + 6)$$

6. Equations with Fractions & Decimals

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Clear fractions or decimals, then solve.

Multiply **EVERY** term on **BOTH** sides by the same nonzero number.

FRACTIONS: USE THE LEAST COMMON DENOMINATOR

$$\frac{x}{2} + 3 = \frac{x}{4} + 5$$

The LCD of **2** and 4 is **4**.

$$4\left(\frac{x}{2}\right) + 4(3) = 4\left(\frac{x}{4}\right) + 4(5)$$

Multiply every term by 4.

$$2x + 12 = x + 20$$

Simplify.

$$x + 12 = 20$$

Subtract **x** from both sides.

$$x = 8$$

Subtract **12** from both sides.



Check: $\frac{8}{2} + 3 = \frac{8}{4} + 5 \rightarrow 7 = 7$

DECIMALS: MULTIPLY BY A POWER OF 10

$$0.5x + 1.2 = 0.2x + 3.6$$

Each decimal has one decimal place.

$$5x + 12 = 2x + 36$$

Multiply **EVERY** term by **10**.

$$3x + 12 = 36$$

Subtract **2x** from both sides.

$$3x = 24$$

Subtract **12** from both sides.

$$x = 8$$

Divide both sides by **3**.



Check: $0.5(8) + 1.2 = 0.2(8) + 3.6 \rightarrow 5.2 = 5.2$



Use **10** for tenths, **100** for hundredths, and **1,000** for thousandths.

Solving equations with fractions and decimals

Q26. Solve for x. Show the operation applied to both sides.

$$\frac{x}{3} + 2 = \frac{x}{6} + 5$$

Q27. Solve for x. Show the operation applied to both sides.

$$0.7x + 1.4 = 0.2x + 4.9$$

Q28. Solve for x. Show the operation applied to both sides.

$$\frac{x+2}{4} = \frac{x-1}{3}$$

Q29. Solve for x. Show the operation applied to both sides.

$$0.25x + 1.5 = 0.1x + 3.6$$

Q30. Solve for x. Show the operation applied to both sides.

$$\frac{2x-1}{3} + 1 = \frac{x+7}{2}$$

7. How Many Solutions?

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Simplify the equation and **interpret** what remains.

1 ONE SOLUTION

$$5x + 7 = 2x + 19$$

$$3x + 7 = 19$$

$$3x = 12$$

$$x = 4$$

Start with the equation.

Subtract $2x$ from both sides.

Subtract 7 from both sides.

Divide both sides by 3 .

$\Rightarrow$ **Exactly one value** makes the equation true.

2 NO SOLUTION

$$3(x + 2) = 3x + 9$$

$$3x + 6 = 3x + 9$$

$$6 = 9$$

FALSE STATEMENT

Start with the equation.

Distribute.

Subtract $3x$ from both sides.

$\Rightarrow$ **No value of x** makes the equation true.

3 INFINITELY MANY SOLUTIONS

$$2(x + 3) = 2x + 6$$

$$2x + 6 = 2x + 6$$

$$6 = 6$$

TRUE STATEMENT

Start with the equation.

Distribute.

Subtract $2x$ from both sides.

$\Rightarrow$ **Every real value of x** makes the equation true.



When variable terms cancel, check whether the remaining statement is **true** or **false**.

How many solutions?

Q31. Classify the equation as one solution, no solution, or infinitely many solutions. If there is one solution, find it.

$$4x + 3 = 2x + 15$$

Q32. Classify the number of solutions. Explain the statement left when the variables cancel.

$$5(x + 2) = 5x + 13$$

Q33. Classify the number of solutions. Explain why checking only one value is not a proof.

$$4(x - 3) + 5 = 4x - 7$$

Q34. Simplify and classify. Do not divide by zero.

$$2(3x + 1) - x = 5x + 8$$

Q35. Simplify both sides completely and determine the solution set.

$$3(2x - 4) + 2(x + 5) = 4(2x - 1) + 2$$

8. Word Problems & Checking Solutions

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Define a variable, write an equation, solve, and **check**.

1 WHEN DO TWO PLANS COST THE SAME?

Bike rental Plan A charges \$12 plus \$5 per hour.
Plan B charges \$24 plus \$3 per hour. After how many hours do the plans cost the same?



Plan A:
\$12 + \$5 per hour

Plan B:
\$24 + \$3 per hour

1 DEFINE THE VARIABLE

Let h be the number of rental hours.

2 WRITE THE EQUATION

$$12 + 5h = 24 + 3h$$

Set the two total costs equal.

3 SOLVE

$$12 + 2h = 24$$

$$2h = 12$$

$$h = 6$$

Subtract $3h$ from both sides.

Subtract 12 from both sides.

Divide both sides by 2.

4 CHECK IN THE ORIGINAL EQUATION

$$12 + 5(6) = 24 + 3(6)$$

$$42 = 42$$

Plan A: \$12 + \$30 = \$42

Plan B: \$24 + \$18 = \$42

⇒ After 6 hours, both plans cost \$42.



State the answer with units and **check** that it makes sense in the story.

Modeling word problems and checking solutions

Q36. Scooter Plan A charges \$14 plus \$4 per hour. Plan B charges \$26 plus \$2 per hour. After how many hours do the costs match, and what is that cost?

Q37. Maya has saved \$18 and adds \$6 each week. Leo has saved \$42 and adds \$3 each week. After how many weeks will they have equal savings?

Q38. Two identical boxes each contain $3x + 2$ markers. A large box contains $4x + 20$ markers. Together, the two identical boxes hold as many markers as the large box. Find x and the number of markers in each small box.

Q39. Two printing services charge \$12 plus \$1.75 per poster and \$24 plus \$1.25 per poster. For how many posters do the prices match, and what is the shared cost?

Q40. A kayak rental costs \$30 for the first hour plus \$8 for each additional hour. A second rental costs \$18 plus \$10 for every hour. For a whole-number rental of at least one hour, when do the costs match?

Homework review

R1. Solve using equal operations on both sides.

$$7x + 2 = 3x + 26$$

R2. Combine like terms, then solve.

$$5x + 3 - 2x + 4 = x + 19$$

R3. Collect the variables on one side and solve.

$$3x - 8 = 7x + 12$$

R4. Distribute, simplify, and solve exactly.

$$4(x + 2) - 3 = 2(x + 10)$$

Homework review

R5. Explain the sign of each distributed term and solve.

$$11 - 2(x - 3) = 3x + 2$$

R6. Clear the fractions and solve.

$$\frac{x+1}{3} + 2 = \frac{x}{2}$$

R7. Determine whether there is one solution, no solution, or infinitely many solutions.

$$6(x - 2) + 5 = 6x - 7$$

R8. Game Center A charges \$9 admission and \$5 per game. Center B charges \$21 admission and \$3 per game. For how many games do the totals match?

Homework word problems

W1. Nina has 6 stickers and gets 2 more each day. Omar has 12 stickers and gets 1 more each day. After how many days will they have the same number?

W2. A club shirt order costs \$10 for setup plus \$3 per shirt at one shop. Another shop charges \$22 for setup plus \$1 per shirt. How many shirts make the totals equal?

W3. Ava starts with 25 points and earns 4 points per round. Ben starts with 10 points and earns 7 per round. After how many rounds will their scores be equal?

W4. Delivery Company A charges \$16 plus \$5 per box. Company B charges \$40 plus \$2 per box. For how many boxes do they charge the same, and what is that charge?

Homework word problems

W5. A rectangle has width 9 cm and length x cm. Its perimeter equals the perimeter of a triangle whose sides are x cm, 14 cm, and 16 cm. Find x and the common perimeter.

W6. Three equal packets each contain $x + 4$ beads. Adding 5 loose beads gives the same total as two boxes containing $x + 13$ beads each. Find x and the number of beads in each packet.

W7. Photo Lab A charges \$20 plus \$2.50 per print. Lab B charges \$35 plus \$1.75 per print. How many prints make the bills equal, and what is the bill?

W8. Two alternative ribbon plans use the same original supply of x meters. Plan A uses one-third of the supply plus 18 meters. Plan B uses one-half of the supply plus 8 meters. Both plans use the same amount. Find the original supply and the amount used by either plan.

Homework word problems

W9. An equipment rental package costs \$50 for the first two hours and \$6 for each additional hour. A second package costs \$20 plus \$9 for every hour. For a whole-number rental of at least two hours, when do the costs match?

W10. A school orders three boxes containing $x + 2$ notebooks each and two packs containing x notebooks each. Another order has four boxes containing $x + 5$ notebooks each plus one loose notebook. The totals match. Find x and the total notebooks in each order.

W11. Print Shop A charges an \$18 setup fee plus \$2.40 per poster, then applies a \$6 discount to the entire bill. Shop B charges \$8 plus \$2.80 per poster. Find the number of posters at which the final bills match and the cost.

W12. A workshop offers Plan A: \$60 covers two participants, each additional participant costs \$7, and a \$4 coupon reduces the whole bill. Plan B charges a \$24 booking fee plus \$9 for every participant. For a whole-number group of at least two people, find the group size at which costs match. Then determine which plan is cheaper for 15 participants and by how much.