SCHOOL OF MATH

PRE-ALGEBRA

CLASS 18

Solving and Graphing One-Variable Inequalities

Compare • Solve • Graph

$< \quad > \quad \leq \quad \geq$

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1. Understanding Inequality Symbols

An inequality compares two values that may not be equal.

$<$	**Less than** The value on the left is smaller than the value on the right.	$3 < 7$
$>$	**Greater than** The value on the left is greater than the value on the right.	$9 > 4$
$\leq$	**Less than or equal to** The value on the left is smaller than or equal to the value on the right.	$x \leq 5$ means x can be 5 or any smaller number.
$\geq$	**Greater than or equal to** The value on the left is greater than or equal to the value on the right.	$x \geq 2$ means x can be 2 or any larger number.

Remember:

The open side faces the greater value.
The pointed end faces the smaller value.

$3 < 7$

↑ smaller ↑ greater

Try it: 6 __ 9 8 __ 3

Answers: <, >

Understanding inequality symbols

Q1. Write < or > in each blank: 12 __ 15, 20 __ 8, and −3 __ −5.

Q2. Translate each sentence into symbols: “x is less than or equal to 9” and “y is greater than −2.”

Q3. Which of the values 1, 4, and 6 make the statement x ≥ 4 true?

Q4. Decide whether each statement is true or false, and explain: 10 ≤ 10 and 10 < 10.

Q5. Write n > 15 in words. Then give two numbers that make it true and one number that makes it false.

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2. Checking Solutions

A solution makes an inequality true when you substitute it for the variable.

1 Substitute
Replace the variable with the given value.

2 Simplify
Do the arithmetic on the left side.

3 Decide: true or false
Determine whether the statement is true or false.

Is $x = 4$ a solution of $x + 3 < 9$?

$$4 + 3 < 9$$
$$7 < 9$$

TRUE
— Yes, $x = 4$ is a solution.

Is $x = 6$ a solution of $x + 3 < 9$?

$$6 + 3 < 9$$
$$9 < 9$$

FALSE
— No, $x = 6$ is not a solution.

Watch the boundary!

For $x \leq 5$, $x = 5$ works because $5 \leq 5$ is true.

For $x < 5$, $x = 5$ does not work because $5 < 5$ is false.

Try it: Which values satisfy $2x \geq 8$?

$x = 3$ $x = 4$ $x = 6$

Answers: $x = 4$ and $x = 6$

Remember: An inequality can have many solutions.

Checking solutions

Q6. Is x = 5 a solution of the inequality?

$$x + 4 < 10$$

Q7. Is x = 7 a solution? Pay attention to the boundary.

$$3x \geq 21$$

Q8. Is x = 2 a solution?

$$5x - 1 > 9$$

Q9. Which of the values x = 1, x = 3, and x = 5 satisfy the inequality?

$$2x + 1 \leq 7$$

Q10. Is x = −4 a solution? Be careful with the negative numbers.

$$x - 2 < -5$$

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3. Graphing Inequalities on a Number Line

A graph shows all the values that make an inequality true.

Open circle: < or >

○ | Boundary not included.

Closed circle: ≤ or ≥

● | Boundary included.

$x < 3$

Open circle at 3. Shade **LEFT.**



$x \leq 3$

Closed circle at 3. Shade **LEFT.**



$x > 3$

Open circle at 3. Shade **RIGHT.**



$x \geq 3$

Closed circle at 3. Shade **RIGHT.**



Remember:

Less than → shade left.
Greater than → shade right.
The arrow means the solutions continue without end.

Graphing inequalities on a number line

Q11. Graph the inequality on a number line.

$x < 2$



Q12. Graph the inequality on a number line.

$x \geq -1$



Q13. Graph the inequality on a number line.

$x \leq 4$



Q14. Graph the inequality on a number line.

$x > -3$



Q15. Write the inequality shown by the graph.



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4. Solving Inequalities Using Addition and Subtraction

Isolate the variable by doing the same operation to both sides.

Remember: Adding or subtracting the same number on both sides keeps the inequality sign unchanged.

Subtract to undo addition

$$x + 4 < 9$$
$$x + 4 - 4 < 9 - 4$$
$$x < 5$$

Subtract 4 from **BOTH** sides.

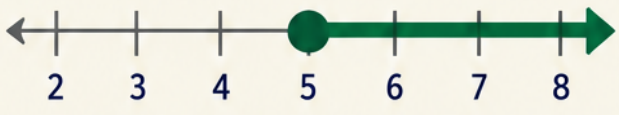


Open circle at 5; shade left.

Add to undo subtraction

$$x - 3 \geq 2$$
$$x - 3 + 3 \geq 2 + 3$$
$$x \geq 5$$

Add 3 to **BOTH** sides.



Closed circle at 5; shade right.

Check a solution: For $x < 5$, try $x = 4$:
$4 + 4 < 9 \rightarrow 8 < 9$ ✓

Try it: $x + 6 \leq 10$ $x - 7 > 1$

Answers: $x \leq 4$; $x > 8$

Solving inequalities using addition and subtraction

Q16. Solve and graph the solution.

$x + 5 < 12$



Q17. Solve and graph the solution.

$x - 6 \geq 3$



Q18. Solve and graph the solution.

$x + 9 \leq 4$



Q19. Solve and graph the solution.

$x - 2 > -7$



Q20. Solve and graph the solution. Notice that the variable is on the right side.

$14 \leq x + 8$



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5. Solving Inequalities Using Multiplication and Division

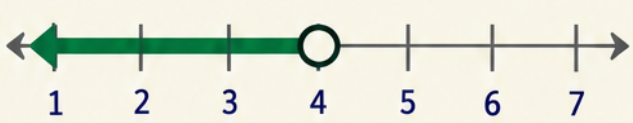
Use inverse operations to isolate the variable.

Remember: When you multiply or divide both sides by a **POSITIVE** number, keep the inequality sign unchanged.

Divide to undo multiplication

$$3x < 12$$
$$3x \div 3 < 12 \div 3$$
$$x < 4$$

Divide **BOTH** sides by positive 3.

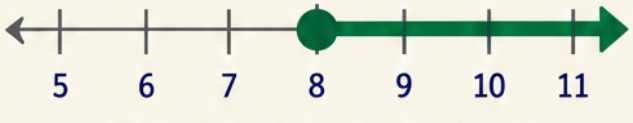


Open circle at 4; shade left.

Multiply to undo division

$$\frac{x}{4} \geq 2$$
$$\frac{x}{4} \times 4 \geq 2 \times 4$$
$$x \geq 8$$

Multiply **BOTH** sides by positive 4.



Closed circle at 8; shade right.

Check a solution: For $3x < 12$, try $x = 2$:
$3(2) < 12 \rightarrow 6 < 12$ ✓

Try it: $5x \leq 20$ $\frac{x}{3} > 2$

Answers: $x \leq 4$; $x > 6$

Solving inequalities using multiplication and division

Q21. Solve and graph the solution.

$4x < 20$



Q22. Solve and graph the solution.

$\frac{x}{5} \geq 3$



Q23. Solve and graph the solution.

$6x \leq 42$



Q24. Solve and graph the solution.

$\frac{x}{2} > -3$



Q25. Solve and graph the solution. The boundary is not a whole number.

$2x \leq 9$



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6. Reversing the Inequality Sign

Multiply or divide BOTH sides by a NEGATIVE number → reverse the inequality sign.

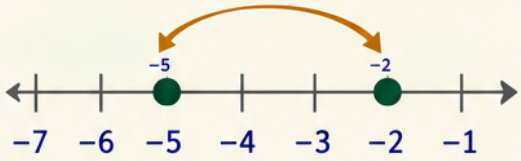
Remember: | $<$ becomes $>$ $\quad$ $\leq$ becomes $\geq$

Why does the sign reverse?

Start with $2 < 5$. Multiply both sides by -1:

$$-2 > -5$$

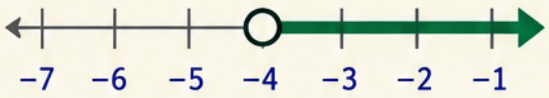
The order reverses on the number line.



Divide by a negative number

$$-3x < 12$$
$$(-3x) \div (-3) > 12 \div (-3)$$
$$x > -4$$

Divide by -3 and **REVERSE** $<$ to $>$.

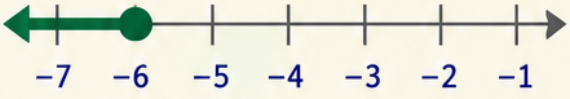


Open circle at -4; shade right.

Multiply by a negative number

$$\frac{x}{-2} \geq 3$$
$$\left(\frac{x}{-2}\right) \times (-2) \leq 3 \times (-2)$$
$$x \leq -6$$

Multiply by -2 and **REVERSE** $\geq$ to $\leq$.



Closed circle at -6; shade left.

Remember: | Adding or subtracting a negative number does **NOT** reverse the sign.

$$x - 3 < 5 \rightarrow x < 8$$

Try it: | $-2x \leq 10$ $\quad$ $-x > 4$ | **Answers:** $x \geq -5$; $\quad x < -4$

Reversing the inequality sign

Q26. Solve and graph the solution.

$$-4x < 20$$



Q27. Solve and graph the solution.

$$-x \geq 3$$



Q28. Solve and graph the solution.

$$\frac{x}{-3} \leq 2$$



Q29. Solve and graph the solution.

$$-5x > -15$$



Q30. A student solves this inequality and writes x ≤ −4. Explain the error, give the correct solution, and graph it.

$$-2x \leq 8$$



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7. Solving Two-Step Inequalities

Undo addition or subtraction **first**. **Then** undo multiplication or division.

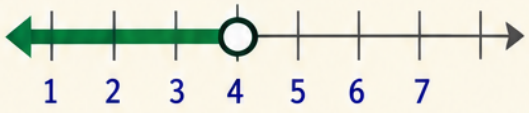
Remember: Do the same operation to **BOTH** sides.

Example 1: Positive coefficient

$$2x + 3 < 11$$

$$2x < 8$$ Subtract 3 from both sides.

$$x < 4$$ Divide both sides by 2.



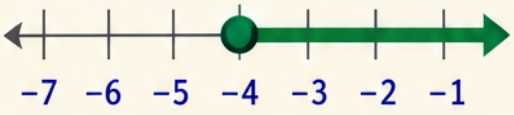
Open circle at 4; shade left.

Example 2: Negative coefficient

$$-3x + 2 \leq 14$$

$$-3x \leq 12$$ Subtract 2 from both sides.

$$x \geq -4$$ Divide by −3 and **REVERSE** the sign.



Closed circle at −4; shade right.

Check a solution:

In $-3x + 2 \leq 14$, try $x = 0$:

$$-3(0) + 2 \leq 14 \rightarrow 2 \leq 14$$ ✓

Remember: Reverse the inequality sign **only** when multiplying or dividing both sides by a negative number.

Try it: $3x - 5 > 7$ $-2x + 1 \geq 9$

Answers: $x > 4$; $x \leq -4$

Solving two-step inequalities

Q31. Solve and graph the solution.

$$3x + 4 < 19$$



Q32. Solve and graph the solution.

$$2x - 7 \geq 3$$



Q33. Solve and graph the solution.

$$-4x + 1 \leq 13$$



Q34. Solve and graph the solution.

$$\frac{x}{3} + 2 > 1$$



Q35. Solve and graph the solution. The variable term is subtracted.

$$5 - 2x < 11$$



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8. Inequality Word Problems

Translate the words. Write an inequality. Solve and interpret.

Common Phrases:	at least	at most	more than	less than
	$\geq$	$\leq$	$>$	$<$

Example 1: A spending limit

You have $26. Admission costs $8, and each ride costs $3. How many rides can you afford?

Let r = number of rides.

$$8 + 3r \leq 26 \quad \text{Subtract 8.}$$
$$3r \leq 18 \quad \text{Divide by 3.}$$
$$r \leq 6$$

Interpretation:

You can afford at most 6 rides.

Since rides are counted, $r = 0, 1, 2, 3, 4, 5,$ or 6.

Example 2: A savings goal

You have $12 and save $4 each week. How many weeks until you have at least $32?

Let w = number of weeks.

$$12 + 4w \geq 32 \quad \text{Subtract 12.}$$
$$4w \geq 20 \quad \text{Divide by 4.}$$
$$w \geq 5$$

Interpretation:

You need to save for at least 5 weeks.

Try it: Each ticket costs $5. You can spend at most $20. How many tickets can you buy?

Answer: $5t \leq 20 \rightarrow t \leq 4$. At most 4 tickets.

Remember: Always explain what your answer means in the situation.

Inequality word problems

Q36. You have $30 to spend at an arcade. Entry costs $6 and each game costs $4. How many games can you play?

Q37. Maya has $15 and saves $5 each week. How many weeks until she has at least $50?

Q38. A freight elevator can carry at most 800 pounds. A worker who weighs 150 pounds loads boxes that weigh 25 pounds each. How many boxes can ride with the worker?

Q39. A phone plan costs $20 per month plus $3 per gigabyte of data. Jordan wants the bill to be less than $41. How many gigabytes can Jordan use?

Q40. A class needs more than $120 for a trip. The class already has $45 and sells cookies for $3 each. How many cookies must the class sell?

Homework review

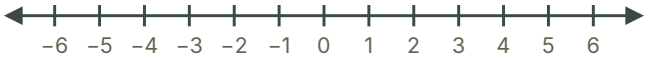
R1. Write < or > in each blank: −8 __ −2, 0 __ −1, and 3.5 __ 3.05.

R2. Is x = 6 a solution? Watch the boundary.

$$2x - 5 \geq 7$$

R3. Graph the inequality on a number line.

$$x > -2$$



R4. Solve and graph the solution.

$$x - 9 \leq -4$$



R5. Solve and graph the solution.

$$\frac{x}{4} < 2$$



Homework review (continued)

R6. Solve and graph the solution.

$-3x \geq 18$



R7. Solve and graph the solution.

$4x - 3 > 13$



R8. Concert tickets cost $7 each. You can spend at most $45. How many tickets can you buy?

Homework word problems

W1. Pencils cost $2 each. You can spend at most $14. How many pencils can you buy?

W2. A roller coaster requires riders to be at least 48 inches tall. Leo is 41 inches tall. How many more inches must Leo grow to ride?

W3. A bus seats 40 passengers, and 26 passengers are already on board. How many more passengers can board?

W4. Ava earns $12 per hour. How many hours must she work to earn more than $96?

W5. A $50 gift card is used to buy books that cost $9 each. How many books can be bought?

Homework word problems (continued)

W6. A taxi charges $4 plus $2 per mile. Sam can spend at most $18. How many miles can Sam ride?

W7. Nina has $28 and saves $6 each week. After how many weeks will she have at least $100?

W8. Diego scored 82 and 90 on two tests. What score on a third test gives him an average of at least 88?

W9. Three times a number decreased by 7 is less than 20. What are the possible values of the number?

W10. The temperature is 10 °C and is dropping 2 °C every hour. After how many hours will the temperature be below −4 °C?

Homework word problems (continued)

W11. A diver is at −15 feet and descends 3 feet per minute. For safety, the diver must stay at or above −45 feet. For how many minutes can the diver keep descending?

W12. Gym A charges a flat $25 per month. Gym B charges $10 per month plus $3 per visit. For how many visits per month is Gym B cheaper than Gym A?