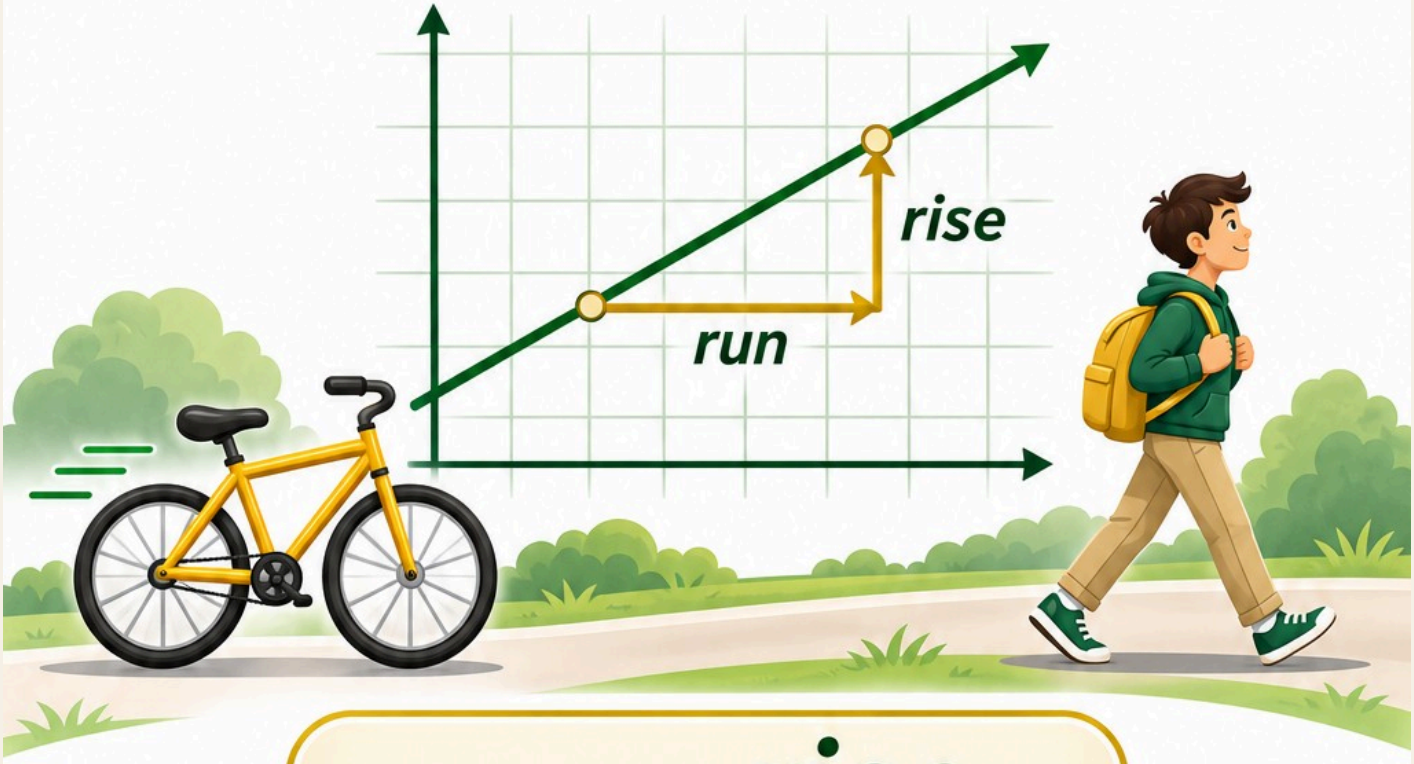
SCHOOL OF MATH

PRE-ALGEBRA | CLASS 20

LINEAR FUNCTIONS

Slope as Rate of Change



$$m = \frac{rise}{run}$$

Tables • Graphs • Real-World Rates

Linear Functions: Slope as Rate of Change

Name: _____ Date: _____

A linear function changes by the same amount for every equal step in its input. In this class you will find rates of change from tables, measure slope as rise over run and with the slope formula, classify slopes, graph lines from a point and a slope, and explain slope as a rate with units.

How to use this workbook

1. Study each teaching image, then answer the five practice questions on the following page. Show your work in the blank space.
2. For every slope, write the change in output over the change in input, and keep the same point order on top and bottom.
3. Read lines from left to right. Use the run as the denominator so each move lands on a grid corner. Blank grids are provided where graphing is required.
4. In word problems, name the input and the output, and always give the slope with units (output units per input unit).
5. Finish with the Homework review (R1–R8) and Homework word problems (W1–W12). Check each answer with a second interval or by substitution.

Contents

1. Understanding linear functions
2. Rate of change from tables
3. Slope: rise over run
4. Slope between two points
5. Types of slope
6. Graphing lines using slope
7. Interpreting slope and its units
8. Applying and comparing rates
9. Homework review
10. Homework word problems

Worked answers for every question are on the class page at schoolofmath.us, behind the blue Answer controls.

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1. Understanding Linear Functions

Equal changes in input produce equal changes in output.



Walking at a steady pace

Each minute, you walk 60 meters.

Time (minutes)	0	1	2	3
Distance (meters)	0	60	120	180
		+60	+60	+60

Constant rate of change: 60 meters per minute.

$$distance = 60 \times time$$

A linear function has a constant rate of change.

Your turn: How far will you walk in 5 minutes?

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Q1

A printer prints 25 pages each minute. Make a table for 0, 1, 2, 3 and 4 minutes, state the rate of change, and write a rule for the number of pages p after t minutes.

Q2

Is the table linear? If it is, state the constant rate of change.

x	0	1	2	3
y	5	9	13	17

Q3

Is this table linear? Explain using the changes in y.

x	0	1	2	3
y	1	2	4	8

Q4

A swimmer swims at a steady 45 meters per minute. Write a rule for the distance d after t minutes. How far does she swim in 7 minutes? How long does 360 meters take?

Q5

Snow on a roof melts steadily. Is the snow depth a linear function of time? If so, give the rate of change with units.

Time (hours)	0	1	2	3
Snow depth (cm)	10	8	6	4

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2. Rate of Change from Tables

Rate of change = change in output ÷ change in input.



Earning money at a steady rate

	+2	+2	+2	
Time (hours)	0	2	4	6
Earnings ($)	0	30	60	90
	+30	+30	+30	

Rate of change = (60 – 30) ÷ (4 – 2)
= 30 ÷ 2 = **$15 per hour**

Divide the *output change* by the *input change*.

Check another interval: 90 ÷ 6 = $15 per hour.

Your turn:

Hours	1	3	5
Earnings ($)	12	36	60

What is the rate of change?

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Q6

Find the rate of change of earnings per hour.

Hours	0	3	6	9
Earnings ($)	0	36	72	108

Q7

A boat rental charges the amounts below. Find the cost per hour.

Hours	2	5	8
Cost ($)	50	125	200

Q8

The inputs in this table are not evenly spaced. Is the rate of change constant? If so, find it.

Minutes	1	4	6	10
Water (liters)	7	28	42	70

Q9

A plant's height is recorded every two days. Find the growth rate. Then explain why $15 \div 6$ gives the wrong answer.

Day	0	2	4	6
Height (cm)	3	7	11	15

Q10

Is this table linear? Check every interval.

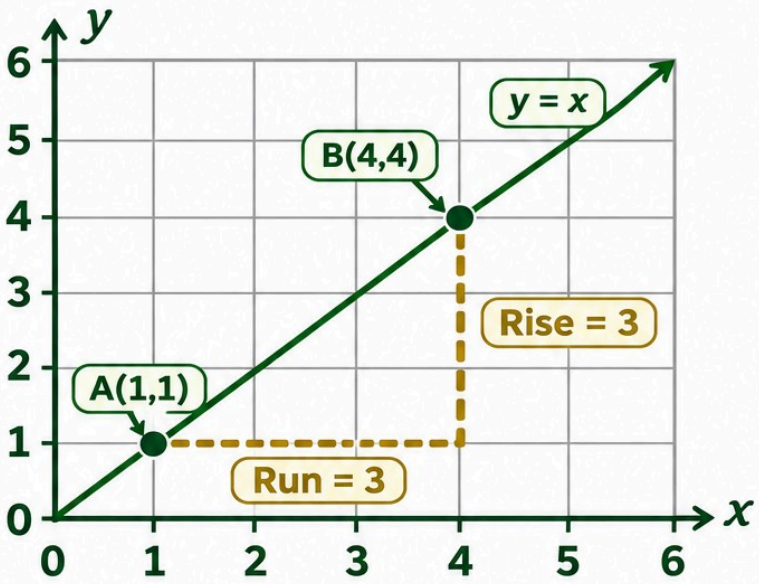
x	1	2	4	5
y	6	10	18	23

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3. Slope: Rise over Run

Slope compares vertical change to horizontal change.



RISE: vertical change (up or down)

RUN: horizontal change (left or right)

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{3}{3} = 1$$

For every 1 unit to the right, the line rises 1 unit.

Your turn: A line rises 4 units and runs 2 units to the right. What is its slope?

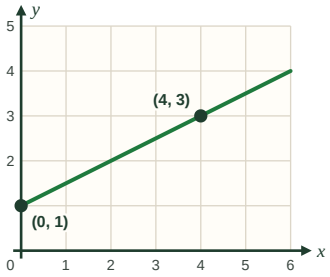
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Q11

A line rises 6 units while it runs 3 units to the right. What is its slope?

Q12

Use rise over run to find the slope of the line.

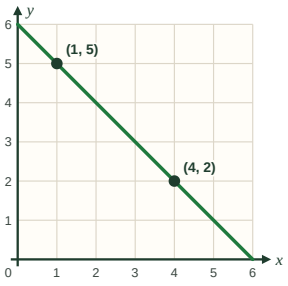


Q13

A line rises 5 units for every 10 units of run. What is its slope? How much does it rise over a run of 4 units?

Q14

Find the slope of the line. Pay attention to the direction of the rise.



Q15

A wheelchair ramp rises 2 feet over a horizontal distance of 24 feet. What is the slope of the ramp? How many feet of run are needed for each 1 foot of rise?

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4. Slope Between Two Points

Find the change in y, then divide by the change in x.



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope through A(2, 3) and B(6, 11).

A(2, 3):	B(6, 11):
$x_1 = 2,\ y_1 = 3$	$x_2 = 6,\ y_2 = 11$

1. **Change in *y*:** $11 - 3 = 8$
2. **Change in *x*:** $6 - 2 = 4$
3. **Divide:** $m = \frac{8}{4} = 2$

Slope = 2: rise 2 units for every 1 unit to the right.

Important Reminder:
Keep the same point order in the numerator and denominator.

$$m = \frac{3 - 11}{2 - 6} = \frac{-8}{-4} = 2$$

Your turn: Find the slope through (1, 4) and (5, 12).

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Q16

Find the slope of the line through (3, 2) and (7, 14).

Q17

Find the slope of the line through (−2, 5) and (4, −7).

Q18

Find the slope of the line through (0, −3) and (6, 1).

Q19

For the points (5, 9) and (1, 1), a student wrote $m = \frac{9-1}{1-5} = -2$. Find the mistake and give the correct slope.

Q20

Find the slope of the line through (−3, 4) and (5, 4). What kind of line is it?

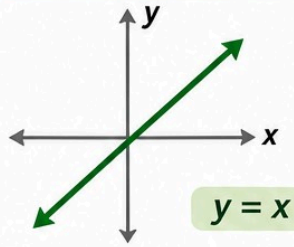
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5. Types of Slope

Read each line from left to right.

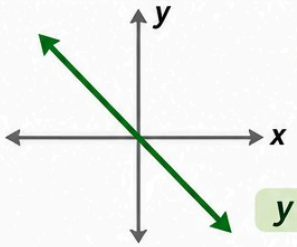
POSITIVE SLOPE



$m > 0$

As x increases, y increases.

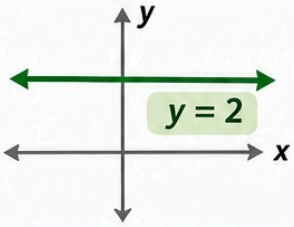
NEGATIVE SLOPE



$m < 0$

As x increases, y decreases.

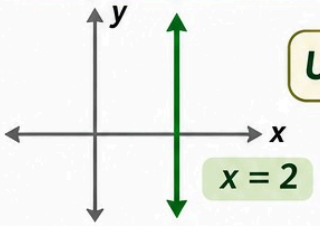
ZERO SLOPE



$m = 0$

No vertical change: rise = 0.

UNDEFINED SLOPE



Undefined

No horizontal change: run = 0.

Horizontal: zero slope. Vertical: undefined slope.

A vertical line is not a function of x.

Your turn: Classify the slopes of $y = 3x$, $y = -2x$, $y = 4$, and $x = 4$.

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Q21

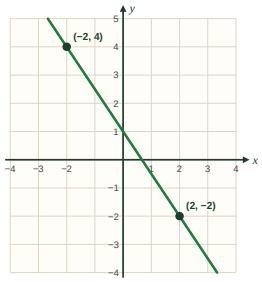
Classify the slope of each line as positive, negative, zero or undefined: (a) a line that goes up from left to right (b) a line that goes down from left to right (c) a horizontal line (d) a vertical line

Q22

Is the slope of the line through (1, 2) and (3, 6) positive or negative? Find its value.

Q23

Is the slope of this line positive or negative? Find its value.



Q24

Classify the slope of each graph, with time as the input: (a) a savings balance when $10 is added every week (b) the gas in a car's tank during a drive (c) the distance from home of a car parked in the garage all day.

Q25

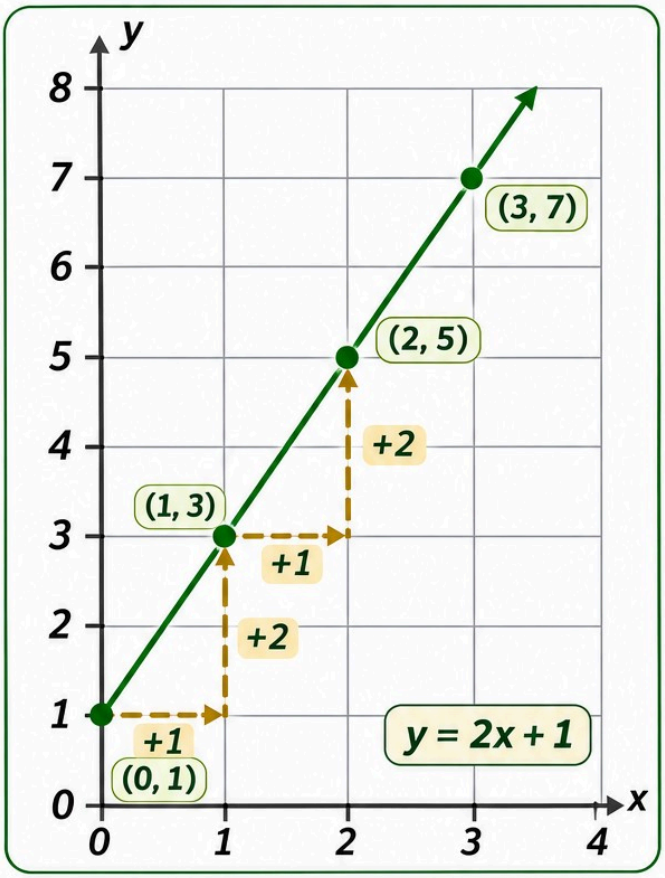
Which of the lines $y = 7$ and $x = 7$ is a function of x? Explain with slope and the vertical-line test.

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6. Graphing Lines Using Slope

Start at a point. Use rise over run to find more points.



EXAMPLE

Start at (0, 1) with slope $m = \frac{2}{1}$.

STEPS

1. *Plot the starting point (0, 1).*
2. *Rise 2 units and run 1 unit right.*
3. *Repeat, then draw a straight line.*

POINTS ON THE LINE

(0, 1) ↓ (1, 3) ↓ (2, 5) ↓ (3, 7)

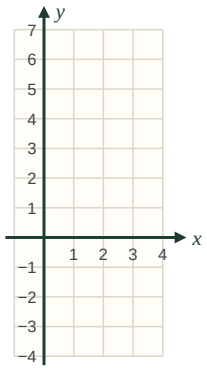
Every move follows the same slope: up 2, right 1.

Your turn: *Start at (0, 2) with slope* $\frac{3}{2}$. *What is the next point when you move right 2 units?*

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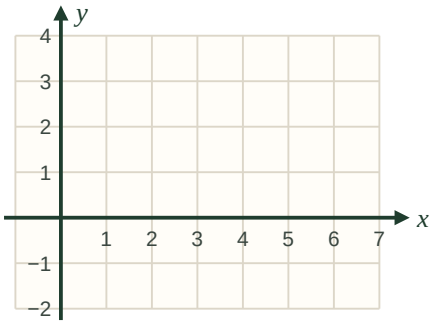
Q26

Start at (0, −3) and use slope 3. List the points for $x = 1, 2, 3$, then graph the line.



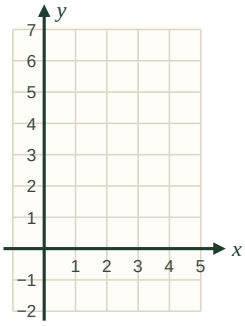
Q27

Start at (0, −1) and use slope $\frac{1}{2}$. Find three more points to the right and graph the line.



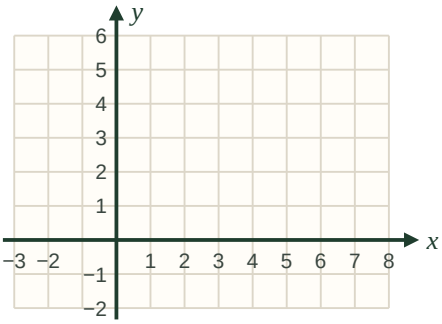
Q28

Start at (0, 6) and use slope −2. Find the points for $x = 1, 2, 3$ and graph the line. Where does it cross the x-axis?



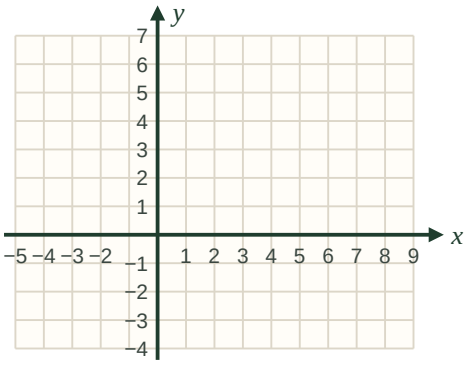
Q29

A line passes through (1, 1) with slope $\frac{2}{3}$. Find two points to the right of (1, 1) and one point to its left.



Q30

Start at (0, 3) with slope $-\frac{3}{4}$. Find the points 4 and 8 units to the right. Is (−4, 6) on the same line?



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7. Interpreting Slope and Its Units

Slope tells how much the output changes for each 1 unit of input.

Slope units = output units ÷ input units

Cycling at a steady speed

Time is the input. Distance is the output.



Time (hours)	Distance (miles)
1	12
2	24
3	36

$$m = \frac{24 - 12}{2 - 1}$$

$$= 12 \text{ miles per hour}$$

For each additional hour, the distance increases by 12 miles.

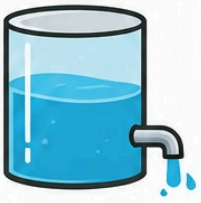
EARNINGS



Slope = 18 dollars per hour

Each extra hour adds $18 to earnings.

DRAINING A TANK



Slope = −3 liters per minute

Each minute, the water volume decreases by 3 liters.

Always name the units and explain what the rate means.

Your turn: *A candle's height has a slope of −2 centimeters per hour. What does this mean?*

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Q31

A phone plan's total cost is shown. Find the slope with units and explain what it means.

Months	1	2	3
Total cost ($)	35	50	65

Q32

A car's fuel tank is recorded during a road trip. Find the slope with units and explain it.

Hours driven	0	2	4
Fuel (gallons)	14	10	6

Q33

A puppy's weight graph has slope 0.25 kilograms per week. What does this mean? How much weight does the puppy gain in 8 weeks?

Q34

A school fundraiser graphs dollars raised (output) against tickets sold (input). The slope is 8. Give the units of the slope and interpret it. How much more money do 15 extra tickets raise?

Q35

An elevator descends at a steady rate. Find the slope with units, interpret it, and find when the elevator reaches the ground (height 0).

Time (s)	0	5	10
Height (m)	60	45	30

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8. Applying and Comparing Rates

Find each slope, use the same units, then compare.

Which cyclist travels faster?





CYCLIST A — TABLE

Time (hours)	Distance (miles)
0	0
2	24
4	48

$$m = \frac{24 - 0}{2 - 0}$$

$$= 12 \text{ miles per hour}$$

CYCLIST B — EQUATION

$$d = 15t$$

d = distance in miles; t = time in hours

The coefficient of t is the rate.

$$m = 15 \text{ miles per hour}$$

15 > 12: Cyclist B travels 3 miles per hour faster.

After 3 hours

Cyclist A: $12 \times 3 = 36$ miles

Cyclist B: $15 \times 3 = 45$ miles

Cyclist B travels 9 miles farther.

Compare rates in matching units.

Reminder: *For a graph, find rise ÷ run between two points.*

Your turn: *Walker A travels 8 miles in 2 hours. Walker B follows $d = 3t$, with miles and hours. Who walks faster, and by how much?*

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Q36

Runner A's distances are in the table. Runner B follows $d = 220t$ (meters, minutes). Who runs faster, and by how much? How far apart are they after 10 minutes?

Time (min)	0	3	6
Distance (m)	0	750	1500

Q37

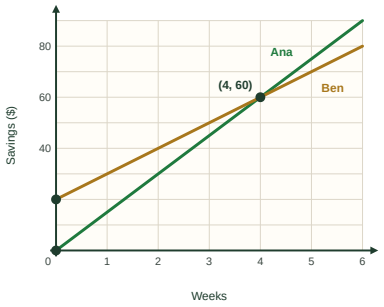
Pump A fills 90 liters in 6 minutes. Pump B follows $V = 18t$ (liters, minutes). Which pump is faster, and by how much?

Q38

Train A travels 2 miles per minute. Train B travels 100 miles per hour. Which is faster? Compare in matching units.

Q39

The graph shows Ana's and Ben's savings. Whose savings grow faster? When do they have the same amount?



Q40

Store A's notebook prices are in the table. Store B charges $C = 1.25n$ dollars for n notebooks. Which store is cheaper per notebook, and how much do you save buying 12 notebooks there?

Notebooks	2	4	6
Cost ($)	3	6	9

R1

Is the table linear? If so, write a rule and find the distance after 6 hours.

Time (hours)	0	1	2	3
Distance (km)	0	8	16	24

R2

Find the rate of change, checking both intervals.

x	2	4	7
y	9	15	24

R3

Line P rises 9 units over a run of 6 units. Line Q falls 4 units over a run of 8 units. Find each slope.

R4

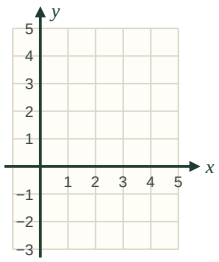
Find the slope of the line through (−1, 7) and (3, −5).

R5

Classify each slope: (a) $y = 0.5x + 3$ (b) $y = -4$ (c) $x = 5$ (d) $y = -2x$

R6

Start at (0, −2) with slope $\frac{3}{2}$. Find the next two points to the right and graph the line.



R7

A pool drains with slope −40 gallons per hour. Explain the slope. If the pool holds 600 gallons, how long does it take to empty?

R8

Train A's distances are in the table. Train B follows $d = 75t$ (miles, hours). Which is faster, by how much, and how far apart are they after 4 hours?

Time (hours)	0	2	5
Distance (miles)	0	160	400

W1

A taxi charges $3 to start plus $2 per mile. Make a table of the total cost for 1, 2, 3 and 4 miles. What is the rate of change, and what does a 10-mile ride cost?

W2

A candle 24 cm tall burns down 1.5 cm per hour. What is the slope of height versus time? How tall is it after 6 hours, and when does it burn out?

W3

The temperature rises steadily from 48 °F at 7 a.m. to 66 °F at 1 p.m. Find the rate of change and the temperature at 10 a.m.

W4

A hiker climbs at a steady rate. After 2 hours she is at 1,400 feet of elevation; after 5 hours she is at 2,600 feet. Find her rate of climb and her starting elevation.

W5

A phone battery drains steadily from 100% to 76% in 60 minutes. Find the rate of change, and find when the battery reaches 20%.

W6

Gym Plan A costs $30 per month with no fee. Plan B costs a $60 joining fee plus $20 per month. Which plan's cost rises faster? After how many months do the plans cost the same?

W7

On a road trip, the distance left to travel is 300 miles at the start and 180 miles after 2 hours. Find the slope with units, interpret it, and find when the family arrives.

W8

A bakery uses 4.5 cups of flour for 3 batches of muffins and 12 cups for 8 batches. Find the flour per batch, and the flour needed for 14 batches.

W9

Lia reads 90 pages in 3 hours. Omar's reading graph is a line through (0, 0) and (4, 100), with hours as input and pages as output. Who reads faster, and how many more pages does that reader finish in 6 hours?

W10

Robot A moves 12 centimeters per minute. Robot B moves 6 meters per hour. Which robot is faster, and by how much in centimeters per minute?

W11

The water in a cooler is 10 liters at 2 hours and 4 liters at 6 hours. Sam says the slope is $\frac{3}{2}$. Find the correct slope and explain Sam's mistake.

W12

Challenge. A ride-share trip costs a fixed fee plus a constant rate per mile. Find the rate per mile and the fixed fee, the cost of a 20-mile ride, and the longest whole-mile ride you can afford with $40.

Miles	2	5	9
Cost ($)	9.50	17.00	27.00