

PROPORTIONAL REASONING



Ratio

Compares two quantities.

$$3:5 \text{ or } \frac{3}{5}$$



Proportion

An equation stating that two ratios are equal.

$$\frac{3}{5} = \frac{6}{10}$$



Read: 3 is to 5 as 6 is to 10.



For $\frac{a}{b} = \frac{c}{d}$, b and d cannot be zero.



EQUIVALENT RATIOS

Two ratios are proportional when they represent the same value.

$$\begin{array}{ccc} & \text{both} & \\ & \text{terms} \times 2 & \\ \frac{3}{5} & \nearrow & \frac{3}{5} = \frac{6}{10} \\ & \searrow & \\ & \text{both} & \\ & \text{terms} \times 3 & \\ & & \frac{3}{5} = \frac{9}{15} \end{array}$$

Multiply or divide the numerator and denominator by the same nonzero number.

$$\frac{3}{5} \neq \frac{4}{6}$$

4/6 simplifies to 2/3, not 3/5.





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TEST 1: SIMPLIFY BOTH

Reduce each ratio to lowest terms.

$$\frac{4}{6} \rightarrow \frac{2}{3} \quad \vdots \quad \frac{6}{9} \rightarrow \frac{2}{3}$$



Same simplified fraction

Therefore, the ratios are proportional.



If the simplified fractions differ,
the ratios are not proportional.



TEST 2: CROSS PRODUCTS

$$\frac{a}{b} = \frac{c}{d}$$

For $\frac{a}{b} = \frac{c}{d}$, compare
the diagonal products.

$$a \times d \text{ and } b \times c$$



Equal cross products →
proportional



Unequal cross products →
not proportional

$$\frac{15}{20} = \frac{9}{12}$$

$$15 \times 12 = 180 = 20 \times 9 = 180$$



Cross-multiplication
clears the denominators.

PROPORTIONAL REASONING



METHOD A: SCALE FACTOR

Use matching numerators or matching denominators.

$$\frac{3}{5} = \frac{x}{20}$$

$$\frac{3}{5} \xrightarrow[\begin{array}{c} 3 \rightarrow x \text{ is } \times 4 \end{array}]{\begin{array}{c} 5 \rightarrow 20 \text{ is } \times 4 \end{array}} \frac{x}{20}$$

Apply the
same factor:

$$\Rightarrow x = 12 \Leftarrow$$



Best when the scale factor is a clean number.



Check: $\frac{12}{20}$ simplifies to $\frac{3}{5}$.

METHOD B: CROSS-MULTIPLY

For $\frac{a}{b} = \frac{c}{d}$, use $a \times d = b \times c$.

1 $\frac{4}{9} \begin{array}{c} \nearrow \searrow \\ \nwarrow \nearrow \end{array} \frac{12}{x}$

2 $4x = 9 \times 12$

3 $4x = 108$

4 $x = 27$

- ✓ Cross-multiplication always works for a true proportion.
- ✓ Use it only when two single fractions are set equal.
- ✓ Denominators cannot be zero.



SETTING UP A PROPORTION

Keep units in the same order on both sides.



$$\frac{\text{quantity A}}{\text{quantity B}} = \frac{\text{quantity A}}{\text{quantity B}}$$

Model

$$\frac{\text{flour}}{\text{cookies}} = \frac{\text{flour}}{\text{cookies}}$$

$$\frac{2}{3} = \frac{x}{12}$$

3 → 12 is $\times 4$

$x = 8$ cups



Getting the setup right is most of the work.

CHECK, CONNECT, ADVANCE

Three fast checks

1.

$$\frac{a}{b} \rightarrow \frac{c}{d}$$

1. Simplify the final ratios.

2.

$$\frac{a}{b} = \frac{c}{d}$$

2. Verify that cross products are equal.

3.

3. Confirm the answer scales in the expected direction.



Common mistake: mixing the unit order.

Proportions lead directly to:

percent • similar triangles • rates
maps • unit conversions • science

A white circle containing a black percent sign (%).

Same ratio. New context.