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Item 1 – Factoring Polynomials

ALGEBRA 2

What is factoring?

Factoring means rewriting a polynomial as a product of simpler expressions.

In simple terms

We **break** a polynomial into factors that multiply together to make the original expression.

Example 1

Factor the polynomial.

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

2 and 3 multiply to 6 and add to 5.

Example 2

Factor out the greatest common factor (GCF).

$$6x^3 + 9x^2 = 3x^2(2x + 3)$$

Both terms share a GCF of $3x^2$.

Why this matters

Factoring helps us simplify expressions, solve equations, and find zeros.

Quick check

Multiply the factors back together to make sure you get the original polynomial.

$x^2 + y^2 = ?$

π

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Item 1A – Greatest Common Factor (GCF)

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What is GCF?

The greatest common factor is the largest factor shared by all the terms of a polynomial.

In simple terms

Find the biggest number and variable part that every term has in common, then factor it out.

Example 1

Factor out the greatest common factor.

$$6x^3 + 9x^2 = 3x^2(2x+3)$$

The GCF is $3x^2$.

Example 2

Find the GCF and factor.

$$8x^2y + 12xy^2 = 4xy(2x+3y)$$

The GCF is $4xy$.

How to find it

1. Find the greatest common factor of the coefficients.
2. Take each shared variable with the lowest exponent.
3. Factor that GCF out of every term.

Quick check

Distribute the factor back to make sure you get the original polynomial.

$x^2+y^2=?$

π

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Item 1B — Factoring Trinomials

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What is a trinomial?

A trinomial is a polynomial with three terms. Factoring a trinomial means rewriting it as a product of simpler binomials.

In simple terms

We look for numbers that multiply to the last term and add to the middle-term coefficient.

Example 1

Factor the trinomial.

$$x^2 + 7x + 12 = (x+3)(x+4)$$

3 and 4 multiply to 12 and add to 7.

Example 2

Factor the trinomial.

$$x^2 + 5x + 6 = (x+2)(x+3)$$

2 and 3 multiply to 6 and add to 5.

How to do it

1. Find two numbers that multiply to *c*.
2. Check that the same two numbers add to *b*.
3. Write the trinomial as two binomials.

Quick check

Multiply the factors back together to make sure you get the original trinomial.

ax^2+bx+c

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Item 1C – Difference of Squares

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What is a difference of squares?

A difference of squares is binomial with two squared terms separated by subtraction.

In simple terms

If both terms are perfect squares and there is a minus sign, we can factor using a special pattern.

Example 1

Factor the expression.

$$x^2 - 25 = (x-5)(x+5)$$

$25 = 5^2$, so this matches $a^2 - b^2$.

Example 2

Factor the expression.

$$9x^2 - 16 = (3x-4)(3x+4)$$

$9x^2 = (3x)^2$ and $16 = 4^2$.

Pattern

$$a^2 - b^2 = (a-b)(a+b)$$

Take the square root of each term. Then write one binomial with a minus sign and one with a plus sign.

Quick check

Multiply the factors back together to make sure you get the original expression.

$$(x-5)(x+5) = x^2 - 25$$

$a^2 - b^2$

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Item 1D – Perfect-Square Trinomials

ALGEBRA 2

What is a perfect-square trinomial?

A perfect-square trinomial is a trinomial that comes from squaring a binomial.

In simple terms

If the first and last terms are perfect squares and the middle term matches $2ab$, the trinomial can be written as a squared binomial.

Example 1

Factor the trinomial.

$$x^2 + 10x + 25 = (x+5)^2$$

$x^2 = x^2$, $25 = 5^2$, and $10x = 2 \cdot x \cdot 5$.

Example 2

Factor the trinomial.

$$x^2 - 6x + 9 = (x-3)^2$$

$x^2 = x^2$, $9 = 3^2$, and $-6x = -2 \cdot x \cdot 3$.

Pattern

$$a^2 + 2ab + b^2 = (a+b)^2$$
$$a^2 - 2ab + b^2 = (a-b)^2$$

Look for two perfect squares and a middle term equal to $\pm 2ab$.

Quick check

Expand the binomial to make sure it matches the original trinomial.

$$(x+5)^2 = x^2 + 10x + 25$$

$x^2 \pm 2xy + y^2$

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Item 1E – Factoring by Grouping

ALGEBRA 2

What is factoring by grouping?

Factoring by grouping is a method used for polynomials with four terms. We group the terms in pairs, factor out the greatest common factor (GCF) from each pair, and then factor out a common binomial factor.

When to use it?

Use factoring by grouping when a polynomial has four terms and there is a common binomial factor after factoring each pair.

It is very common with four terms.

Example 1

Factor the polynomial.

Step 1: Group the terms in pairs.

$$x^3 + 3x^2 + 2x + 6 = (x^3 + 3x^2) + (2x + 6)$$

Group the first two terms and the last two terms.

Step 2: Factor out the GCF from each pair.

$$x^2(x + 3) + 2(x + 3)$$

Factor out x^2 from the first pair and 2 from the second pair.

Step 3: Factor the common binomial.

$$(x + 3)(x^2 + 2)$$

Both terms have the common binomial $(x + 3)$.
Factor it out.

Pattern

For a polynomial with four terms:

$$ax^3 + bx^2 + cx + d$$

1. Group the terms in pairs.
2. Factor out the GCF from each pair.
3. Look for a common binomial factor.
4. Factor it out.

Quick check

Factor the polynomial.

$$2x^3 + 4x^2 + 3x + 6$$

Hint: Group, factor each pair, then look for a common binomial.

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2. Factoring Helps Find Zeros

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Why is this important?

Factoring a polynomial helps us find the **zeros** (also called **roots** or **solutions**) of a function. The zeros are the x-values where the graph crosses the x-axis (i.e., where $f(x) = 0$).

Key Idea

If a polynomial is written in factored form,

$$f(x) = (x - a)(x - b)$$

then the zeros are

$$x = a \quad \text{or} \quad x = b.$$

Example 1

Find the zeros of the function.

Suppose $f(x) = x^2 - 5x + 6.$

Factor: $f(x) = (x - 2)(x - 3).$

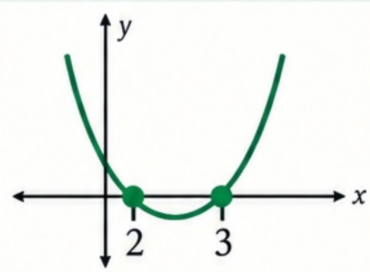
To find the zeros, set the function equal to zero:

$$(x - 2)(x - 3) = 0$$

Therefore: $x = 2$ or $x = 3.$

These are the **roots**, **zeros**, or **solutions**.

Check on the graph



The graph crosses the x-axis at $x = 2$ and $x = 3$.

Key Takeaway

Factors $\longleftrightarrow$ Zeros

The factors of a polynomial give us the zeros of the function, and the zeros tell us where the graph crosses the x-axis.

Quick check

Find the zeros of the function.

$$f(x) = x^2 + 4x - 5$$

Hint: Factor the trinomial, then set each factor equal to zero.

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3. Remainder Theorem

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What is the Remainder Theorem?

The Remainder Theorem says:

When a polynomial $f(x)$ is divided by $x - a$, the remainder is $f(a)$.

So instead of doing polynomial long division, you can simply substitute a.

Key Idea

- If a polynomial $f(x)$ is divided by $x - a$, the remainder is $f(a)$.
- You can find the remainder by evaluating the function at $x = a$.

In symbols:

$f(x) \div (x - a) \longrightarrow$ remainder $= f(a)$

Example 1

Find the remainder.

Suppose

$$f(x) = x^3 + 2x^2 - 5x + 3.$$

What is the remainder when dividing by $x - 2$?

Solution

Step 1: Identify a.
$x - 2 = x - a$, so $a = 2$.

Step 2: Evaluate $f(2)$.

$$\begin{aligned} f(2) &= 2^3 + 2(2^2) - 5(2) + 3 \\ &= 8 + 8 - 10 + 3 \\ &= 9 \end{aligned}$$

Step 3: Conclusion.
The remainder is **9**.

You did not need to perform polynomial division!

Key Takeaway

The Remainder Theorem gives a fast and easy way to find the remainder when a polynomial is divided by $x - a$:

Remainder $= f(a)$

Quick check

Find the remainder when

$$f(x) = x^3 - 4x^2 + x + 6$$

is divided by $x - 3$.

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4. Factor Theorem

ALGEBRA 2

What is the Factor Theorem?

The Factor Theorem says:

$x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

In simple language:

If plugging a into the polynomial gives zero, then $x - a$ is a factor.

Key Ideas

- The Factor Theorem is a special case of the Remainder Theorem.
- If $f(a) = 0$, then $x - a$ is a factor of $f(x)$.
- If $f(a) \neq 0$, then $x - a$ is not a factor of $f(x)$.

In symbols:

$x - a$ **is a factor of** $f(x) \Leftrightarrow f(a) = 0$

Example 1

Check if $x - 2$ is a factor.

Suppose $f(x) = x^3 - 4x^2 + x + 6$.

Is $x - 2$ a factor?

Solution

Step 1: Calculate $f(2)$.

$$
\begin{aligned}
f(2) &= 2^3 - 4(2^2) + 2 + 6 \\
&= 8 - 16 + 2 + 6 \\
&= 0
\end{aligned}
$$

Step 2: Conclusion.

Since $f(2) = 0$, $x - 2$ **is a factor of** $f(x)$.

Key Takeaway

To check if $x - a$ is a factor of a polynomial:

1. Substitute a into the polynomial.
2. If $f(a) = 0$, then $x - a$ is a factor.
3. If $f(a) \neq 0$, then $x - a$ is not a factor.

Quick check

Is $x + 1$ a factor of $f(x) = x^3 + 3x^2 - 4x - 12$?

Hint: Evaluate $f(-1)$. If the result is 0, then $x + 1$ is a factor.

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5. The Sign Can Be Confusing

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What's the issue?

When using the Factor Theorem (or Remainder Theorem), the sign can be confusing. You must use the opposite number of the constant in the factor.

If the factor is $x - a$, you test $f(a)$.
If the factor is $x + a$, you test $f(-a)$.

Key Idea

The Factor Theorem says:

$x - a$ is a factor of $f(x)$ if and only if $f(a) = 0$.

So for a factor of the form $x + a$, rewrite it as $x - (-a)$, and test $f(-a)$.

Factor:	$x - 3$	$\rightarrow$	test $f(3)$
Factor:	$x + 3$	$\rightarrow$	test $f(-3)$
Factor:	$x + 4$	$\rightarrow$	test $f(-4)$

Examples

1 Factor: $x - 3$

Test $f(3)$.

Because $x - 3 = x - a$, we use $a = 3$. So test $f(3)$.

2 Factor: $x + 3$

Rewrite: $x + 3 = x - (-3)$. Test $f(-3)$.

Because $x + 3 = x - (-3)$, we use $a = -3$. So test $f(-3)$.

3 Factor: $x + 4$

Rewrite: $x + 4 = x - (-4)$. Test $f(-4)$.

Because $x + 4 = x - (-4)$, we use $a = -4$. So test $f(-4)$.

Key Takeaway

For a factor $x - a$, test $f(a)$.
For a factor $x + a$, test $f(-a)$.

Always use the opposite number of the constant in the factor.

Quick check

What value should you substitute to check if each is a factor?

1. $x - 5 \rightarrow$ test $f(\ ?\ )$
2. $x + 5 \rightarrow$ test $f(\ ?\ )$
3. $x + 7 \rightarrow$ test $f(\ ?\ )$

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6. Using the Factor Theorem to Completely Factor a Polynomial

ALGEBRA 2

What are we doing?

We use the Factor Theorem to find one factor of the polynomial, then divide by that factor (usually using synthetic division), and continue factoring the resulting polynomial until it is completely factored.

Key Idea

1. If $f(a) = 0$, then $x - a$ is a factor of $f(x)$.
2. Divide by $x - a$ (often using synthetic division).
3. Factor the remaining polynomial.
4. Repeat if needed until completely factored.
5. The zeros are the values of x that make each factor equal to zero.

Example 1

Completely factor the polynomial.

1 Find a factor using the Factor Theorem.

Suppose $f(x) = x^3 - 6x^2 + 11x - 6$.

Try $x = 1$:

$$f(1) = 1 - 6 + 11 - 6 = 0$$

Therefore, $x - 1$ **is a factor of** $f(x)$.

2 Divide by $x - 1$ (synthetic division).

1	1	-6	11	-6
	$\downarrow$	1	-5	6
	1	-5	6	$\boxed{0}$

The quotient is $x^2 - 5x + 6$ and the remainder is 0.

3 Factor the quadratic.

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

Numbers that multiply to 6 and add to -5 are -2 and -3.

4 Write the complete factorization.

$$x^3 - 6x^2 + 11x - 6 = (x - 1)(x - 2)(x - 3)$$

5 Find the zeros.

Set each factor equal to zero:

$x - 1 = 0 \quad x - 2 = 0 \quad x - 3 = 0$

The zeros are $x = 1, 2, 3.$

Key Takeaway

Use the Factor Theorem to find one factor, then divide and factor the rest, until the polynomial is completely factored. The zeros are the values of x that make each factor zero.

Quick check

Completely factor the polynomial and find its zeros.

$$f(x) = x^3 + 2x^2 - 5x - 6$$

Hint: Try $x = 3$ first (or use the Rational Root Theorem).

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7. The Big Algebra 2 Connection

ALGEBRA 2

The Big Picture

These ideas are all connected. For a polynomial, you can go from the expression to its factors, to its zeros, and then to the x-intercepts of its graph.

Polynomial — An algebraic expression in x (e.g., degree 2, 3, ...). → **Factor** — Write the polynomial as a product of simpler factors (e.g., $x - a$). → **Zero** — Set each factor equal to zero and solve for x. → **x-intercept** — Each zero gives an x-intercept on the graph: $(a, 0)$.

Example

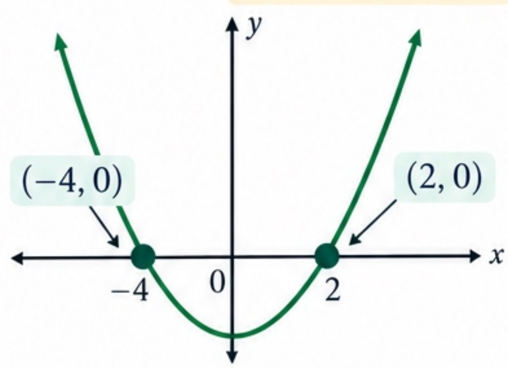
Follow the chain for a specific polynomial.

Suppose $f(x) = (x-2)(x+4)$.

1. **Polynomial** $f(x) = (x-2)(x+4)$
2. **Factors** $x - 2$ and $x + 4$
3. **Zeros** $x - 2 = 0 \rightarrow x = 2$; $x + 4 = 0 \rightarrow x = -4$
4. **x-intercepts** $(2, 0)$ and $(-4, 0)$

Graph

The graph crosses the x-axis at the zeros $x = -4$ and $x = 2$.



Key Takeaway

Factoring, solving polynomial equations, the Factor Theorem, and graphs are all part of the same big idea in Algebra 2.

Polynomial → Factor → Zero → x-intercept

Algebra connects symbols, numbers, and graphs!

Quick check

Consider $f(x) = (x+1)(x-3)$.

1. What are the zeros?
2. What are the x-intercepts?
3. Sketch the graph (opens up or down?).
4. What are the factors?

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